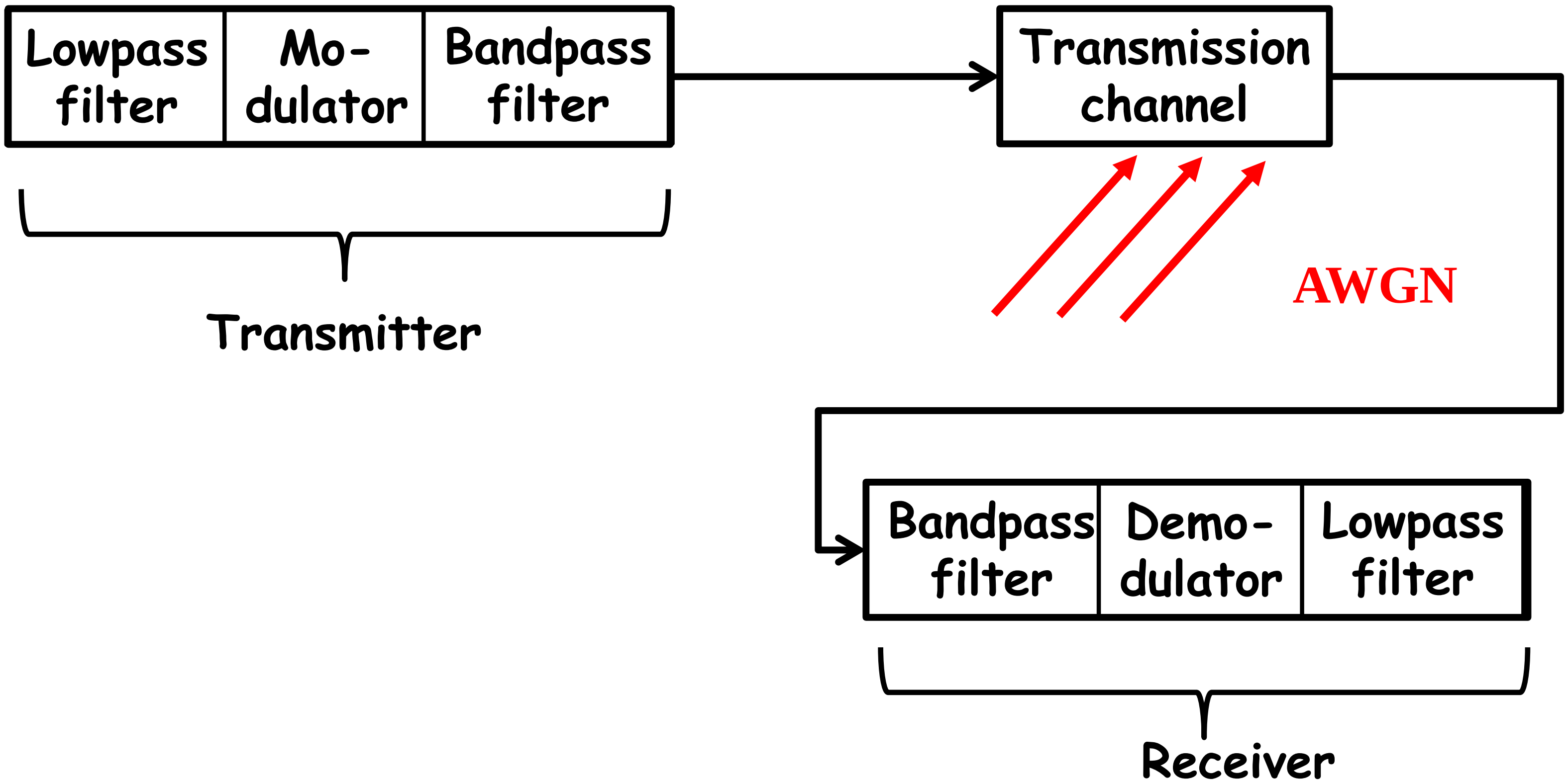
Amplitude & Frequency Modulations (09)

- Telecommunication system
- Definition of modulation
- Spectra of information source signals
- Purposes of using modulation
- The lunch for free does not exist
- Definition of demodulation/detection
- Classification of modulations
- Amplitude Modulation (AM)
- Frequency Modulation (FM)

Telecommunication system



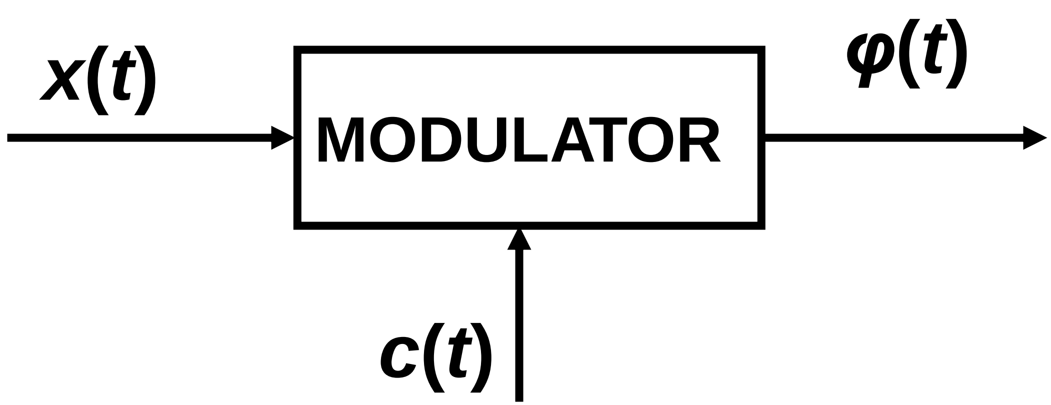
AWGN – Additive White (Wideband) Gaussian Noise

Definition of modulation

Modulation involves three signals:

- **information carrying signal = modulating signal *x*(*t*)**
- **carrier *c*(*t*)**
- **modulated signal *y*(*t*)**

- **Modulation is mapping of information signal *x*(*t*) into one of the parameters of the carrier *c*(*t*).**
- **The output signal *y*(*t*) is called the modulated signal.**

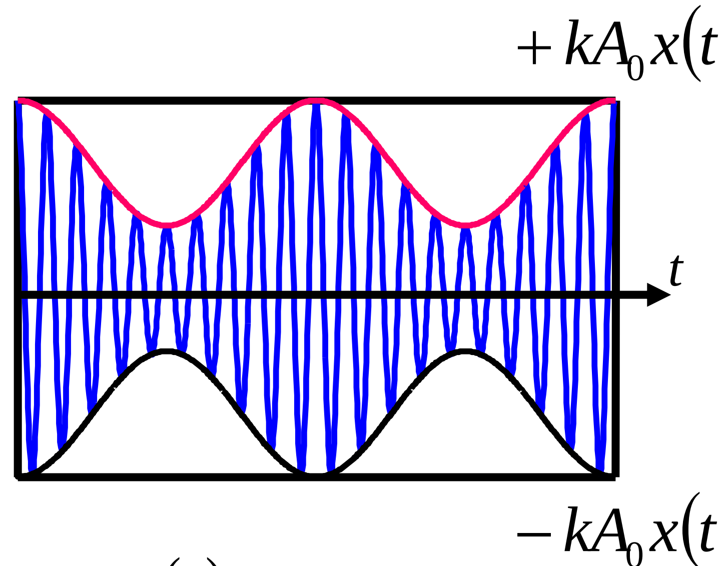


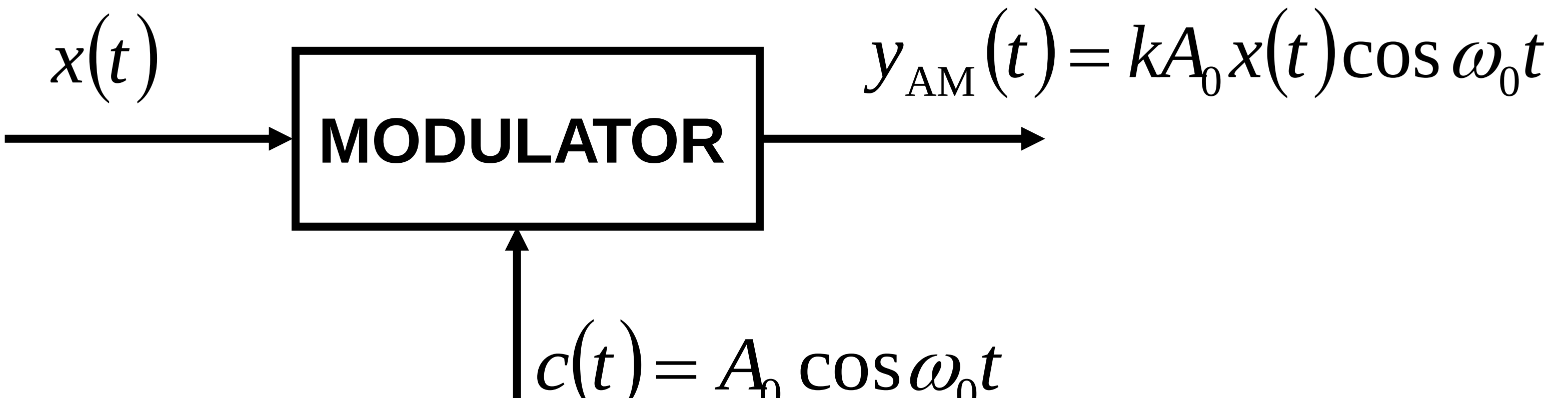
Amplitude Modulation (AM-DSB) (AM-Double SideBand)

carrier : $c(t) = A_0 \cos \omega_0 t, \quad \omega_0 = 2\pi f_0$

modulated signal : $y_{\mathrm{AM}}(t) = kA_0 x(t) \cos \omega_0 t$

k - modulator constant





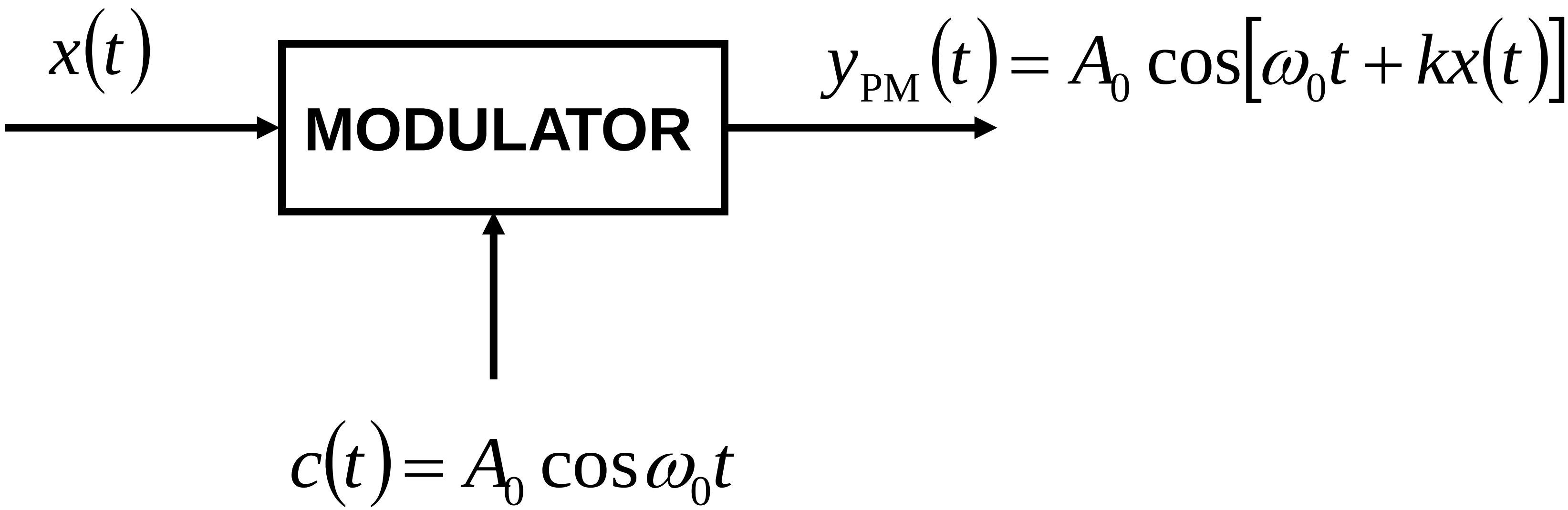
The amplitude $kA_0x(t)$ of the modulated signal fluctuates according to variations of the modulating signal.

Phase Modulation (PM)

$$c(t) = A_0 \cos \omega_0 t, \quad \omega_0 = 2\pi f_0$$

$$y_{\mathrm{PM}}(t) = A_0 \cos[\omega_0 t + kx(t)]$$

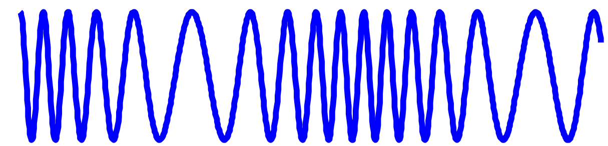
k - modulator constant



Instantaneous phase deviation *kx*(*t*) from the linear trend $\omega_0 t$ of the modulated signal fluctuates according to variations of the modulating signal.

Frequency Modulation (FM)

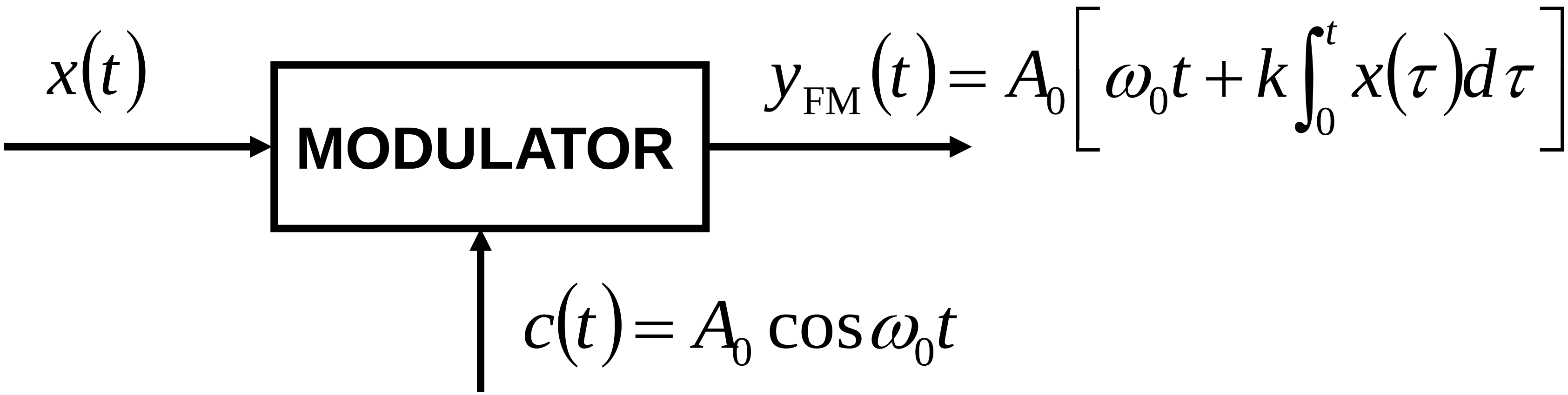
$$c(t) = A_0 \cos \omega_0 t, \quad \omega_0 = 2\pi f_0$$



$$\omega(t) = \omega_0 + kx(t)$$

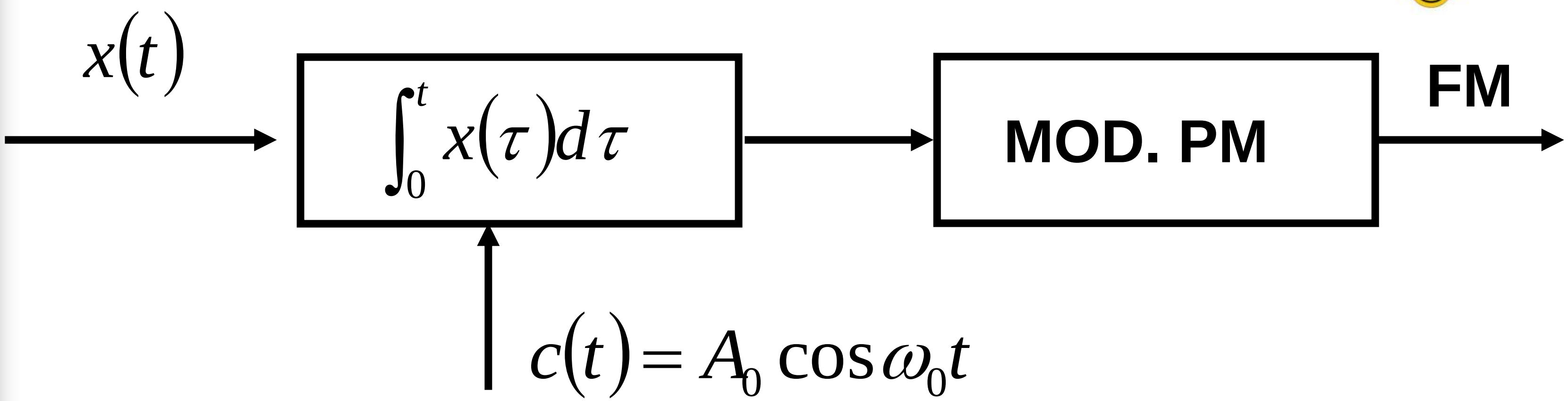
$$y_{\mathrm{FM}}(t) = A_0 \cos\left[\int_0^t \omega(\tau)d\tau\right] = A_0 \cos\left[\omega_0 t + k\int_0^t x(\tau)d\tau\right]$$

k - modulator constant



Instantaneous frequency deviation $kx(t)$ from the carrier frequency ω_0 of the modulated signal fluctuates according to variations of the modulating signal.

Frequency Modulation vs Phase Modulation



1. **Prove that the PM modulator can produce the FM signal provided the information signal gets preintegrated.**

2. **Is it possible to force the FM modulator to produce the PM signal?**

3. **What are your conclusions in terms of spectral analysis of either PM or FM modulations?**

Spectra of information source signals

Voice - baseband – up to several kHzs

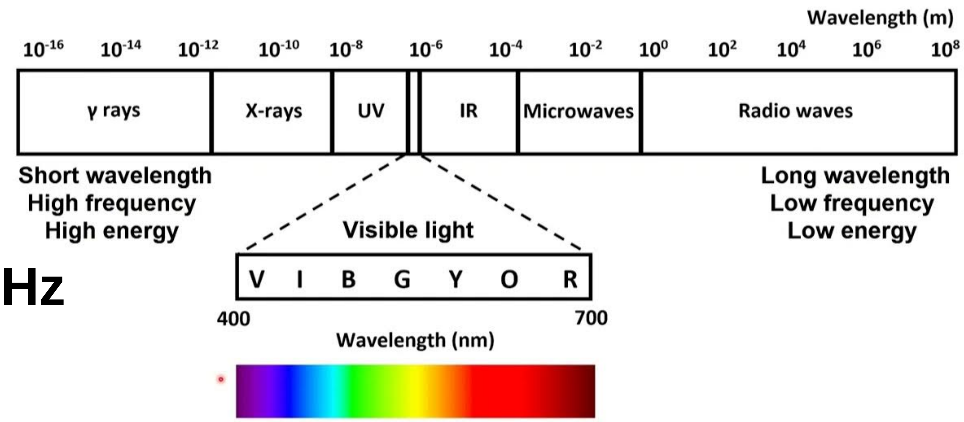
Sound - baseband – up to tens of kHzs

Video – baseband – up to several MHzs

Raw data – baseband – up to ...?

Purposes of using modulation

1. Access to different parts of electromagnetic spectrum



Microwave - MHz

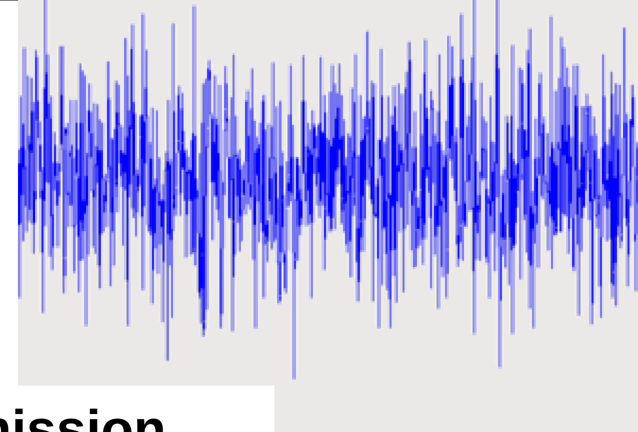
Satellite - GHz

Copper plant – baseband up to 10 MHz

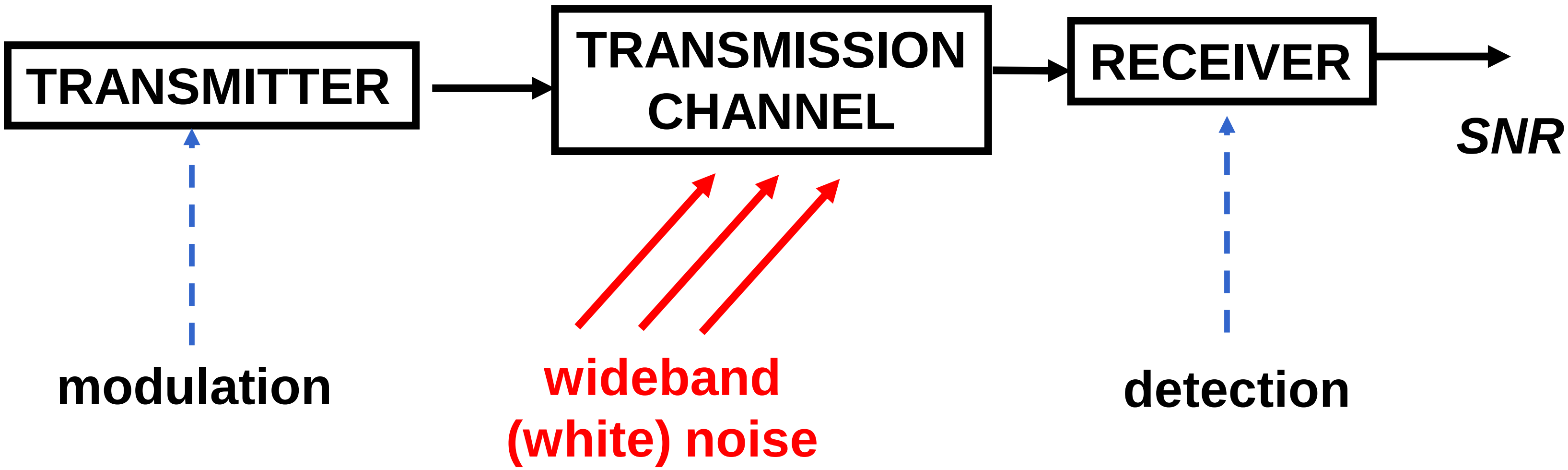
Fiber – narrow frequency window at THz

albo zmieniono jego nazwę. Sprawdź, czy łącze wskazuje poprawny plik i lokalizację.

Purposes of using modulation



2. Improvement noise immunity of transmission

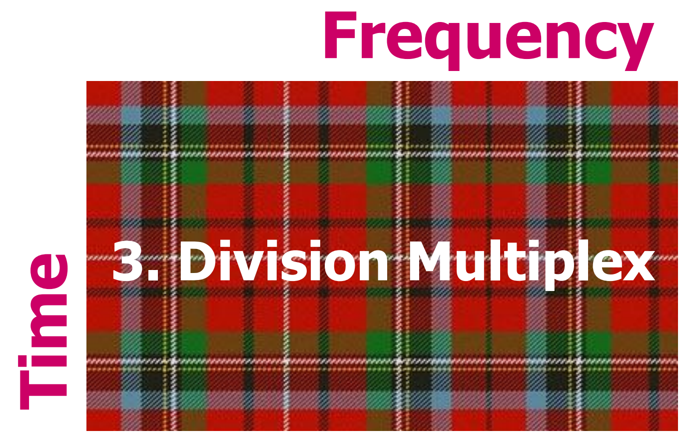


improvement of the Signal to Noise Ratio (*SNR*)

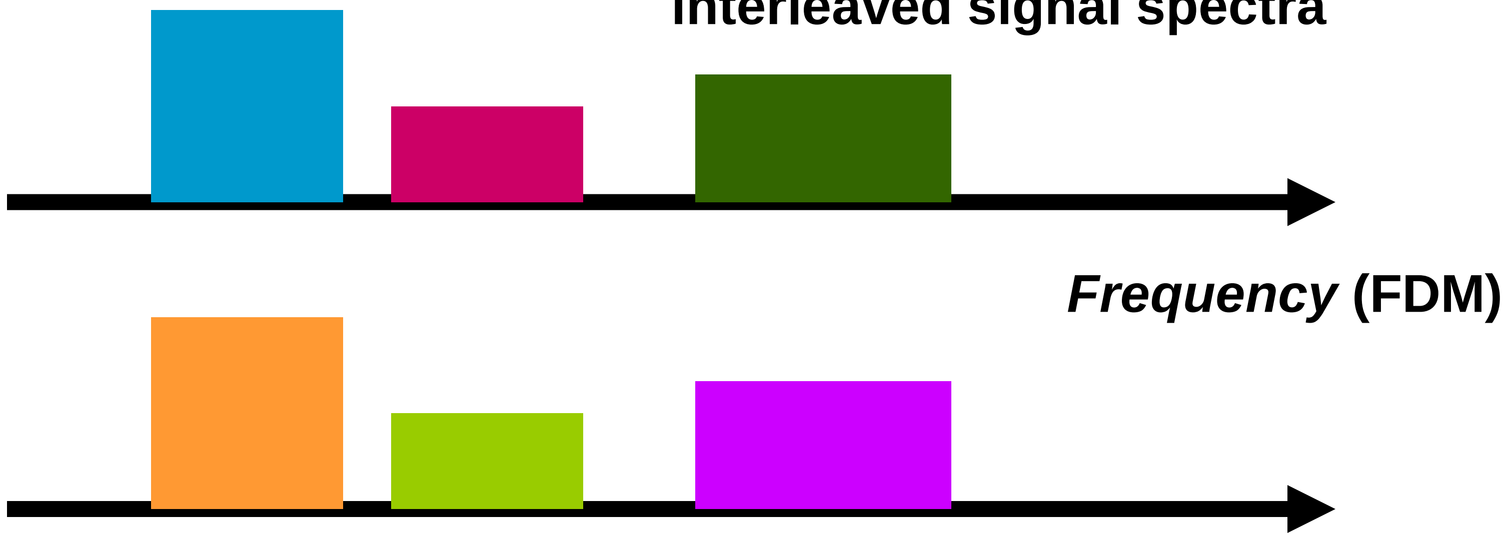
***SNR* (modulation) > *SNR* (baseband system)**

Purposes of using modulation

3. Simultaneous transmission of several signals in a channel



interleaved signal spectra



interleaved data blocks

The lunch for free does not exist

#1 Access to different parts
of the electromagnetic spectrum

#2 Better noise immunity of transmission

#3 Frequency and/or Time Division Multiplex
of transmission channels

#4 SPECTRUM WIDENING
(all modulations increase the bandwitdh)

- **B [Hz] or W [rad/s] – bandwidth of the modulated signal**
- **f_{m} [Hz] or ω_{m} [rad/s] – bandwidth of the modulating signal**

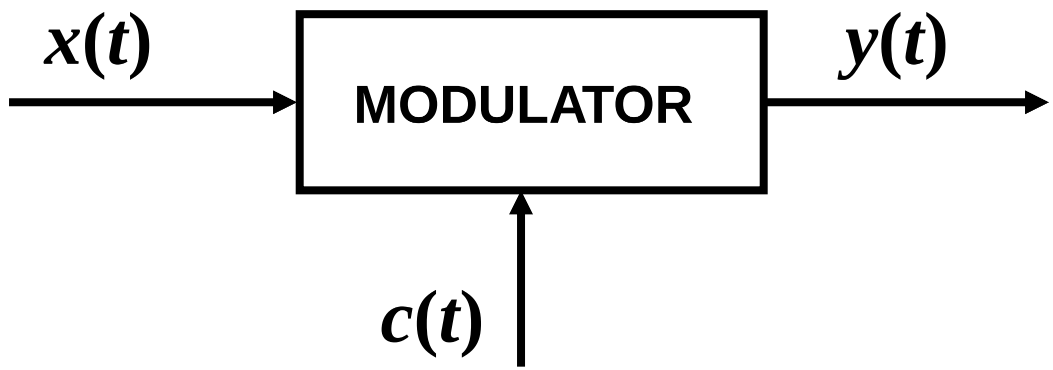
$$W > \omega_{\mathrm{m}} \quad \left(B > f_{\mathrm{m}}\right)$$

$$\kappa = \frac{W}{\omega_{\mathrm{m}}} = \frac{B}{f_{\mathrm{m}}} > 1$$

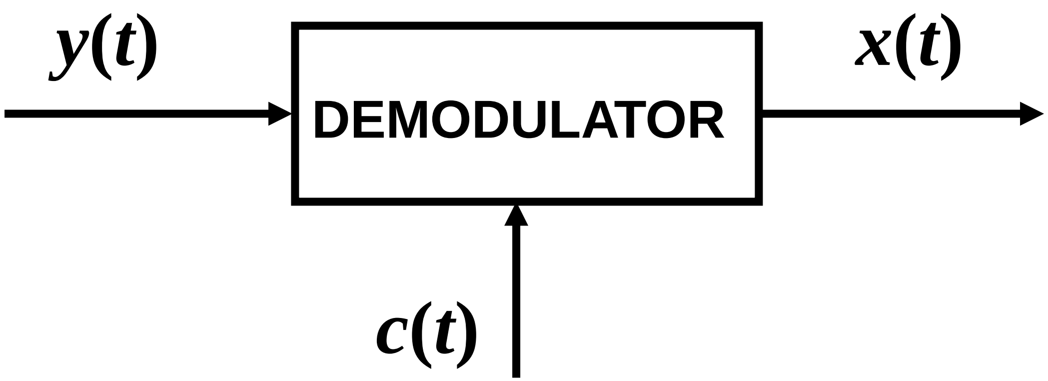
bandwidth widening
(expansion) coefficient

Definition of Demodulation

Modulation is <u>mapping</u> the information signal onto the carrier.

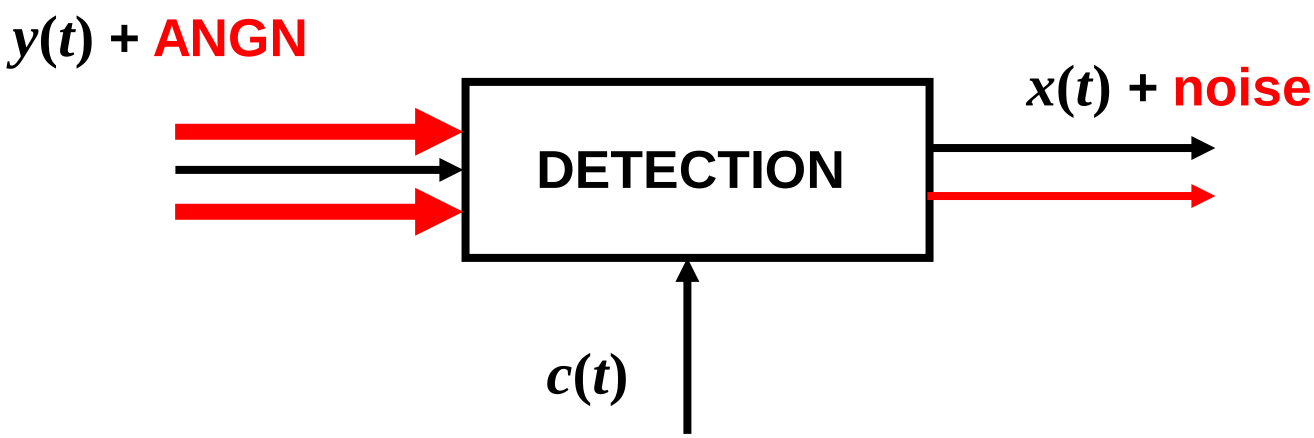


Demodulation is <u>stripping</u> the information signal from the modulated carrier (an inverse operation to modulation).

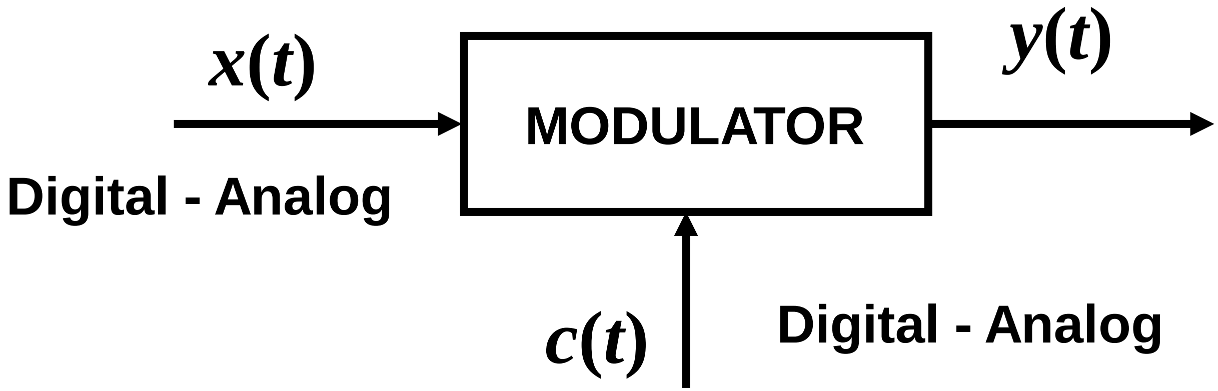


Definition of Detection

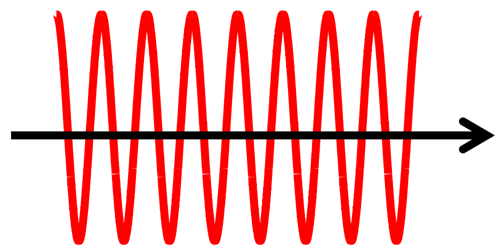
Detection is STRIPPING the information signal from the modulated carrier in a noisy environment (detection = demodulation of signal corrupted by ANGN).



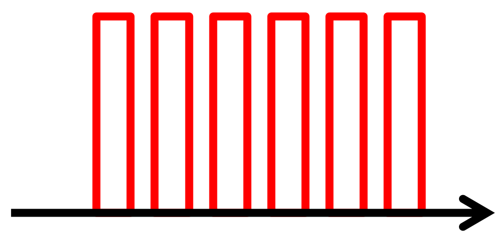
Classification of Modulations



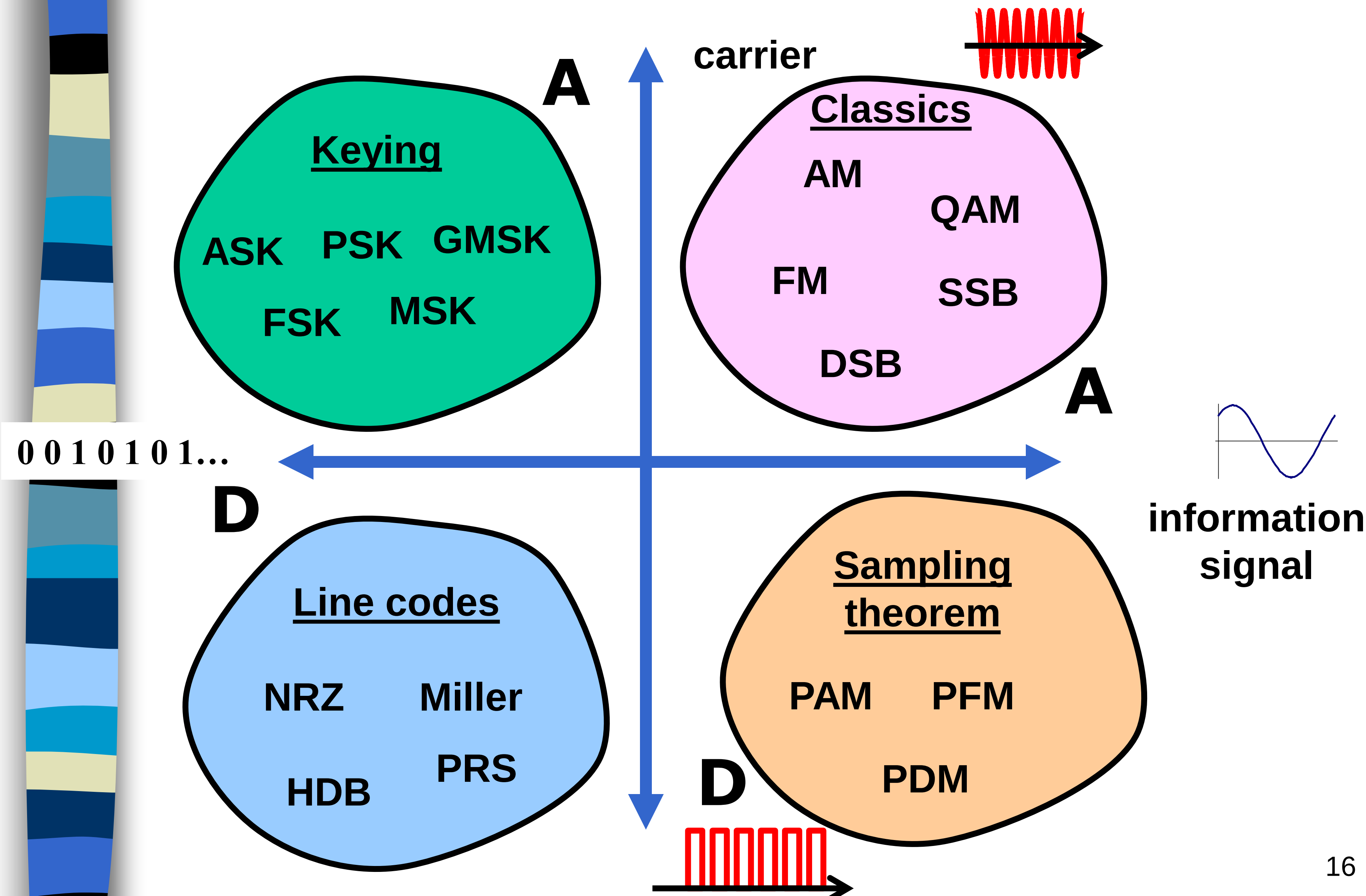
Analog waveform
sin or cos waveform (A)



Digital waveform
sequence of pulses (D)



Classification of Modulations



Amplitude Modulation (AM)

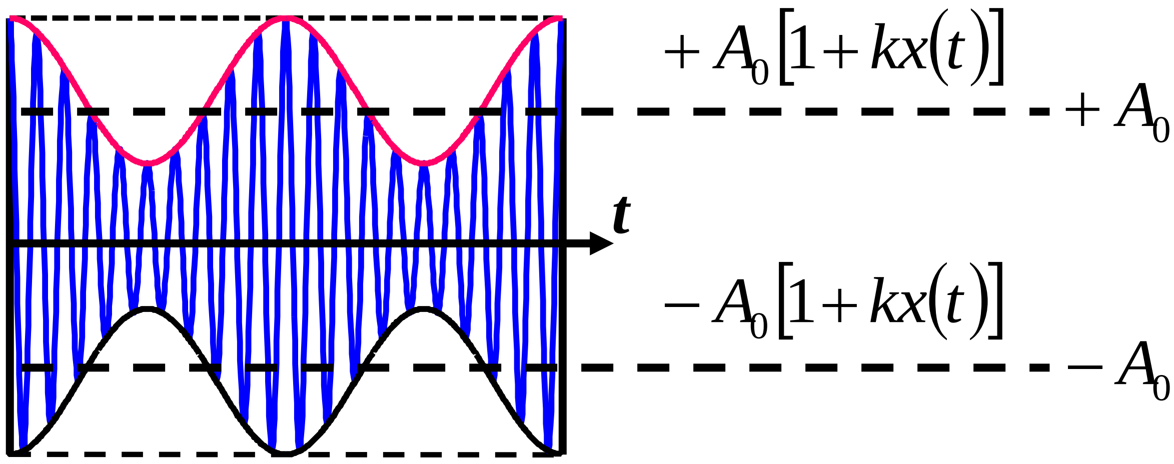
$$y_{\mathrm{AM}}(t) = \underbrace{kA_0 x(t)\cos\omega_0 t}_{\substack{\textbf{Modulation (AM-DSB)} \\ \textbf{(information part of AM)}}} + \underbrace{A_0 \cos\omega_0 t}_{\substack{\textbf{Unmodulated carrier} \\ \textbf{(for envelope detection)}}}$$

$$\boxed{y_{\mathrm{AM}}(t) = A_0\left[1 + kx(t)\right]\cos\omega_0 t}$$

$1 + kx(t) \geq 0$ **no overmodulation for envelope detection**

Amplitude Modulation (AM)
(no overmodulation: 1 + *kx*(*t*) ≥ 0

$$y_{AM}(t) = A_0[1 + kx(t)]\cos\omega_0 t$$

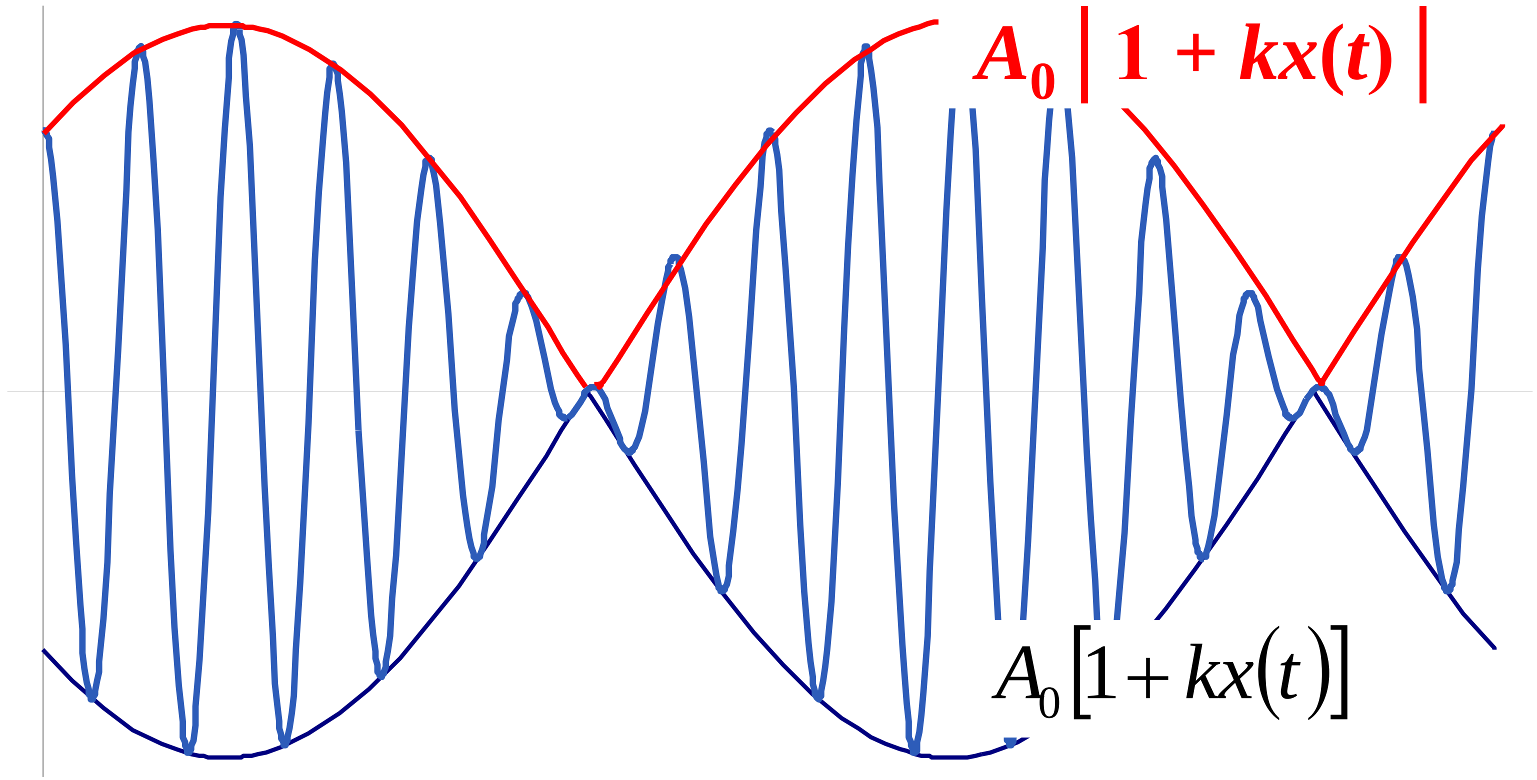


Positive peaks of the AM signal do follow the modulating signal. The envelope is defined as a curve joining positive (negative) peaks. The „envelope" of AM peaks coincides with the modulating signal.

Note: the envelope can be reproduced in the transmitter even though it is not transmitted.

Amplitude Modulation (AM) (overmodulation: 1 + *kx*(*t*) ≤ 0)

$$y_{\mathrm{AM}}(t) = A_0\left[1 + kx(t)\right]\cos\omega_0 t$$



Neither positive peaks (upper envelope) or negative peaks (lower envelope) of the AM signal do follow the modulating signal.

Amplitude Modulation (AM) – single tone

Single tone modulation: $x(t) = a\cos\omega_m t$

$y_{AM}(t)=A_0[1 + kx(t)] \cos\omega_0 t$

$y_{AM}(t)=A_0(1 + ka \cos\omega_m t) \cos\omega_0 t$

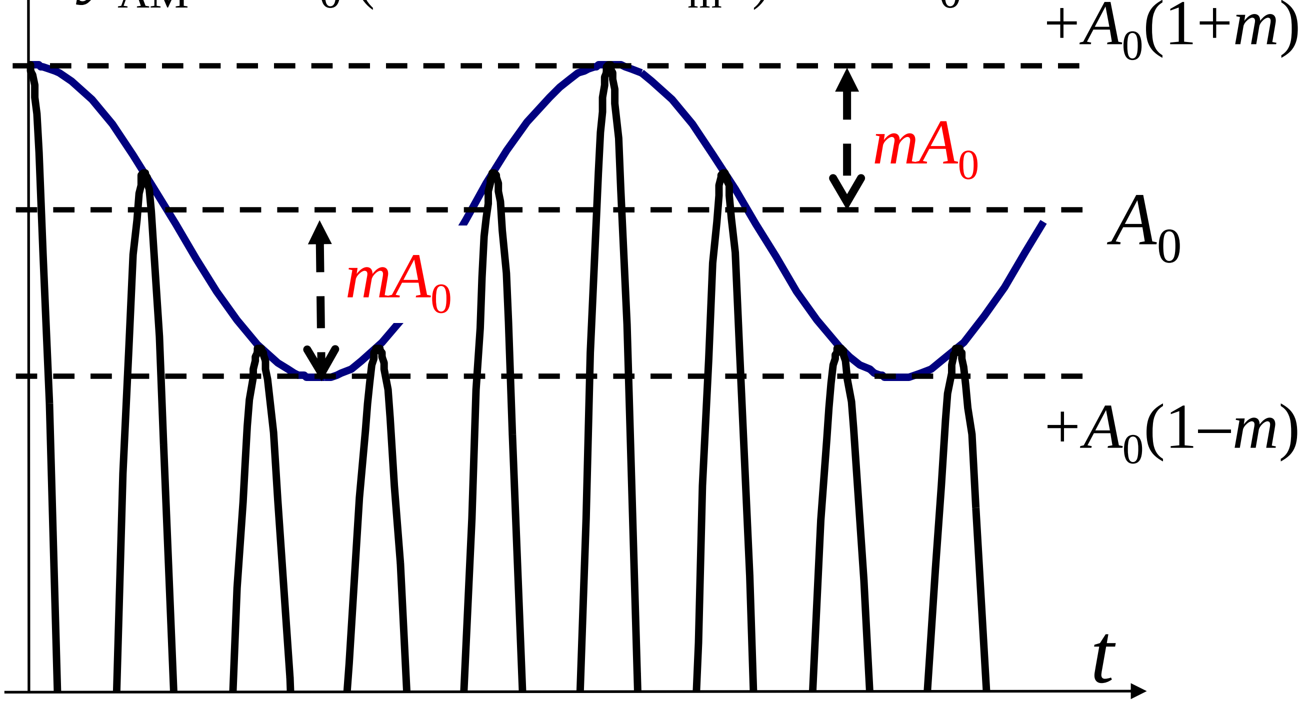
$ka = m$ – modulation (depth) index

$$y_{AM}(t) = A_0(1 + m\cos\omega_m t)\cos\omega_0 t$$

Determine the phasor diagram for the single tone AM signal.

Amplitude Modulation (AM) – single tone

$$y_{\mathrm{AM}} = A_0\left(1 + m\cos\omega_{\mathrm{m}}t\right)\cos\omega_0 t$$

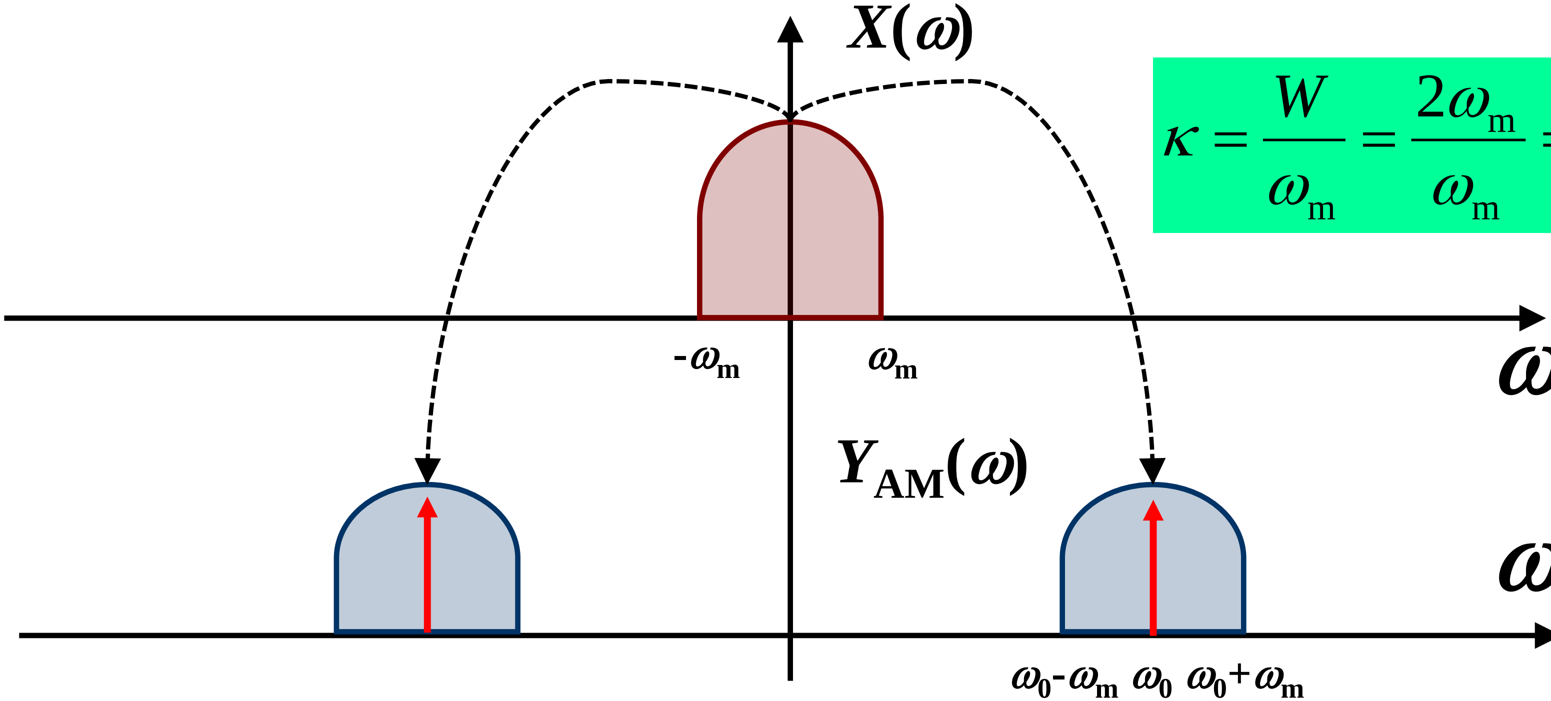


Modulation depth index informs how much the carrier amplitude is changed by the modulating (information) signal. In order to avoid an overmodulation effect the modulation index has to be kept as *m* < 1.

Amplitude Modulation (AM) – spectrum

$$y_{\mathrm{AM}}(t) = A_0[1 + kx(t)]\cos\omega_0 t$$

$$Y_{\mathrm{AM}}(\omega) = \pi A_0 \delta(\omega - \omega_0) + \frac{kA_0}{2} X(\omega - \omega_0), \quad \omega > 0$$



$$\kappa = \frac{W}{\omega_{\mathrm{m}}} = \frac{2\omega_{\mathrm{m}}}{\omega_{\mathrm{m}}} = 2$$

Determine and sketch the spectrum of the single tone AM modulation.

WORK IN PROGRESS

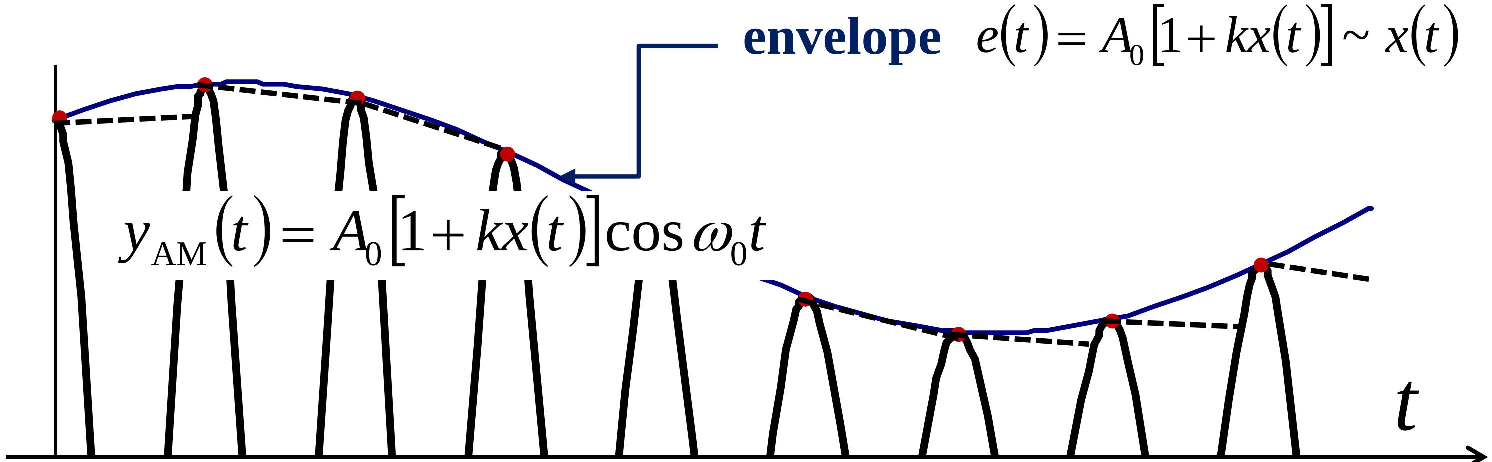
AM Demodulation

Two possible demodulation techniques:

- AM signal - <u>envelope (noncoherent)</u> demodulation (needs transmission of unmodulated carrier)

- AM-DSB - <u>Coherent (synchronous)</u> demodulation (needs synchronization of receiver-transmitter or coherent carrier recovery at a transmitter

AM Envelope (noncoherent) Demodulation



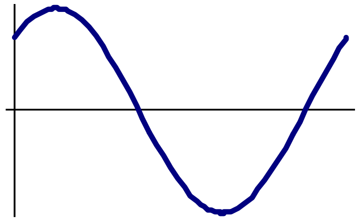
→ **Peak Value Detector (•) & Interpeak Approximation** → **Ideal Low Pass Filter** →

AM Envelope Detector (no carrier recovery at a receiver is necessary)

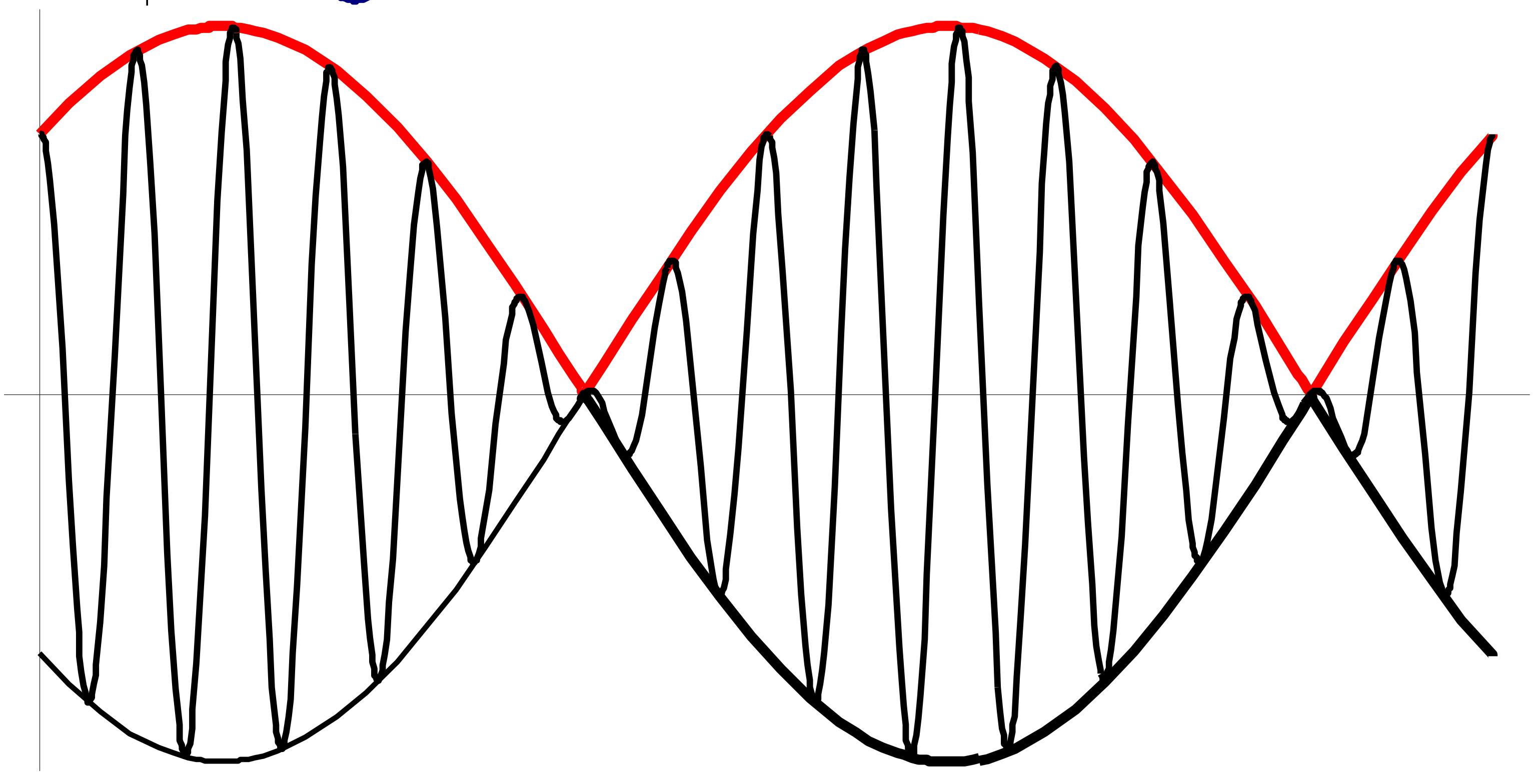
What is the scale of an envelope detection?
Let f_m = 5 kHz and f_0 = 225 kHz
The envelope is „peak sampled" with the frequency f_0 = 225 kHz which is well over the Nyquist sampling frequency $2f_m$ = 10 kHz .

Envelope Detection – overmodulation errors

$1 + kx(t) < 0$



$\longrightarrow \quad e(t) = A_0 \left| 1 + kx(t) \right|$

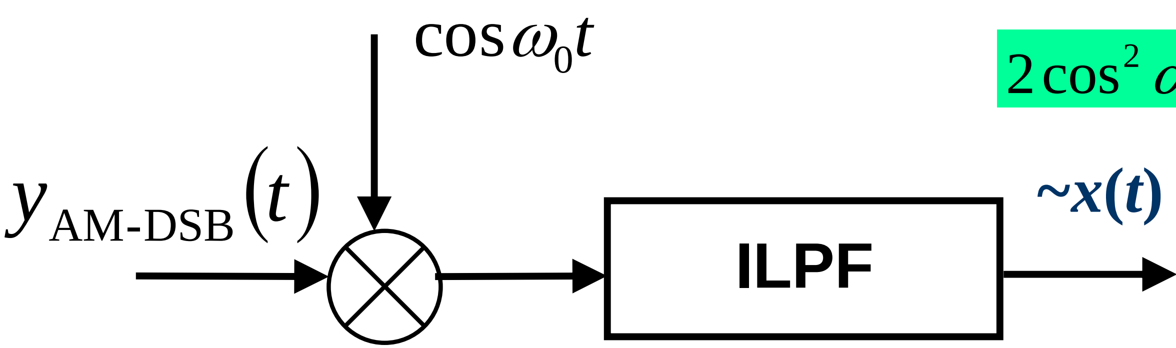


Overmodulation contributes to detection errors as the peak detector detects the absolute value of the envelope $A_0 \left| 1 + kx(t) \right|$ rather than the envelope itself $A_0[1 + kx(t)]$.

AM-DSB Coherent (synchronous) Detection

$$y_{\text{AM-DSB}}(t) = kA_0 \cos \omega_0 t$$

$$2\cos^2 \alpha = 1 + \cos 2\alpha$$



$$\underbrace{y_{\text{AM-DSB}}(t) \times \cos \omega_0 t}_{\text{coherent detection}} = kA_0 x(t) \cos^2 \omega_0 t =$$

$$\frac{1}{2} kA_0 x(t) + \underbrace{\frac{1}{2} kA_0 x(t) \cos 2\omega_0 t}_{\text{ILPF out}} \rightarrow \sim x(t)$$

Ilustrate a coherent detection in the frequency domain (provide suitable graphs with shifted spectra).

Coherent AM-DSB Detection – phase errors

Find the output signal of a coherent receiver provided that the carrier is reproduced with a phase error Δ*φ*.

$$2\cos\alpha\cos\beta=\cos(\alpha-\beta)+\cos(\alpha+\beta)$$

$$\underbrace{\underbrace{\left[x(t)\cos\omega_0 t\right]}_{\text{amplitude modulation}}\times\cos(\omega_0 t+\Delta\varphi)}_{\text{coherent detection}}\sim$$

$$\sim x(t)\cos\Delta\varphi+\underbrace{x(t)\cos(2\omega_0 t+\Delta\varphi)}_{\text{ILPF out}}\rightarrow$$

$$\rightarrow\sim x(t)\cos\Delta\varphi$$

Synchronous operation of both transmitter and receiver is necessary.

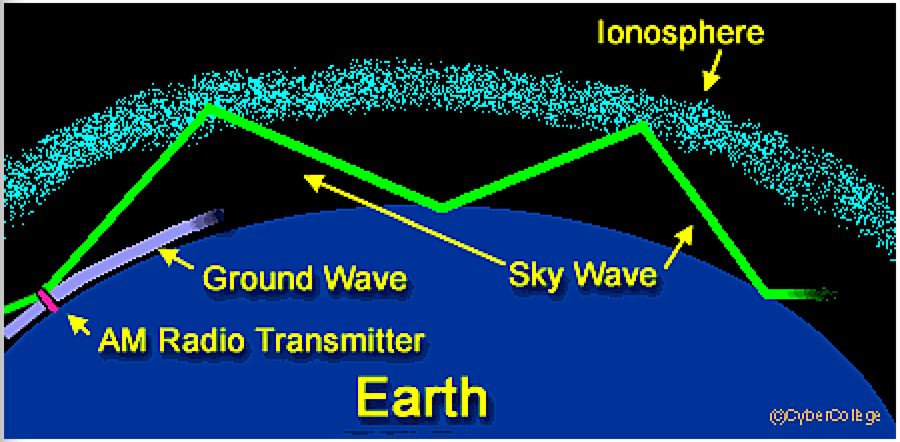
Longwave Transmitter (Solec Kujawski)

The longwave transmitter Solec Kujawski is a longwave broadcasting facility of the Polish Radio mainly for the AM-LW (long wave) radio broadcasting at the 225 kHz carrier frequency with a bandwidth of 9 kHz.

- **Longwave transmitter 1.2 MW max power**
- **Two masts of 330 m and 289 m height**

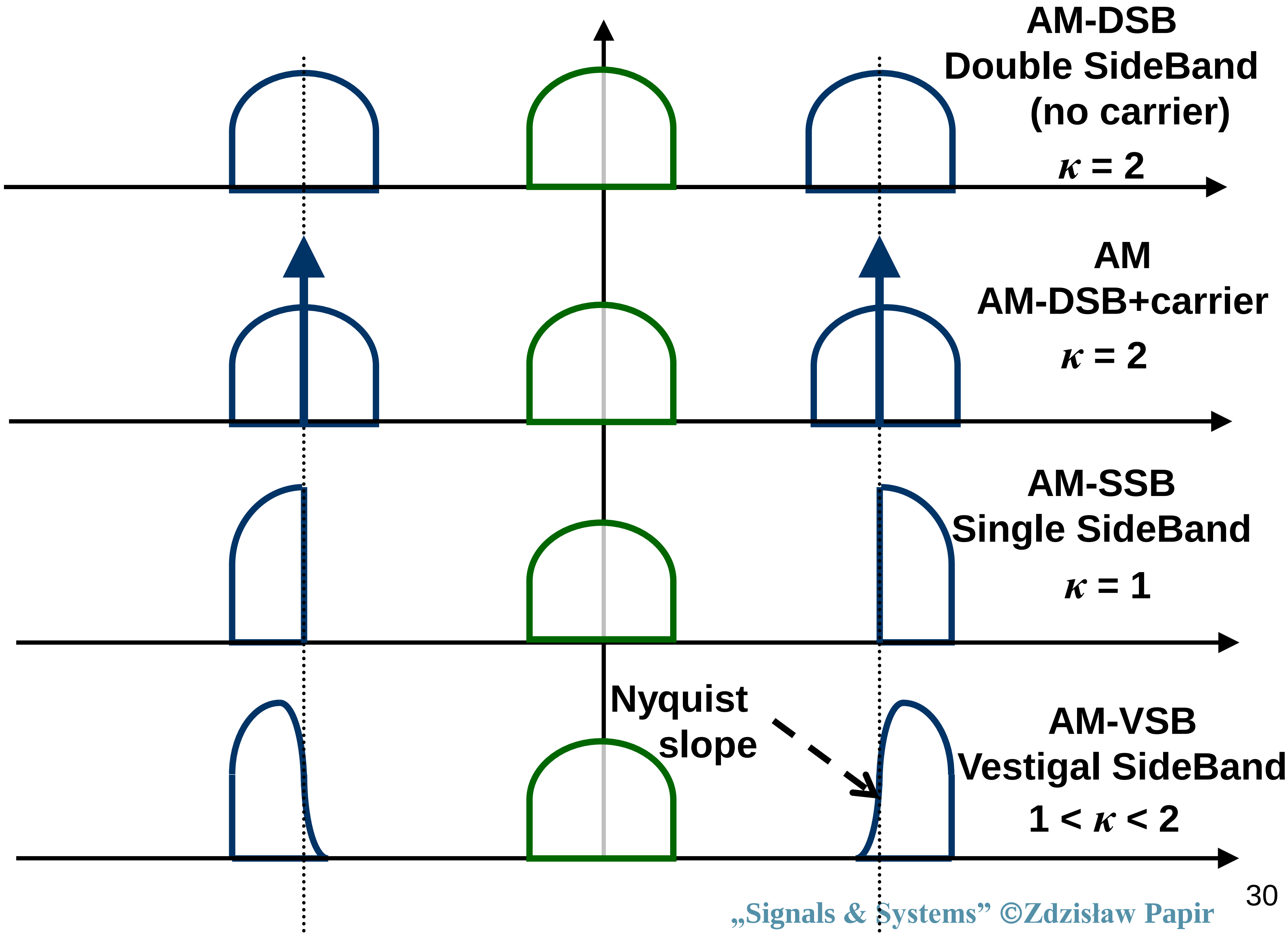
AM Emissions
(radio broadcasting mainly)

LONG WAVES:
30 kHz ÷ 300 kHz (10 km ÷ 1 km)
AM Emissions:
144 kHz ÷ 288 kHz
Distance covered:
1000 km ÷ 2000 km
Propagation:
ground wave (incl. line of sight)
skywave



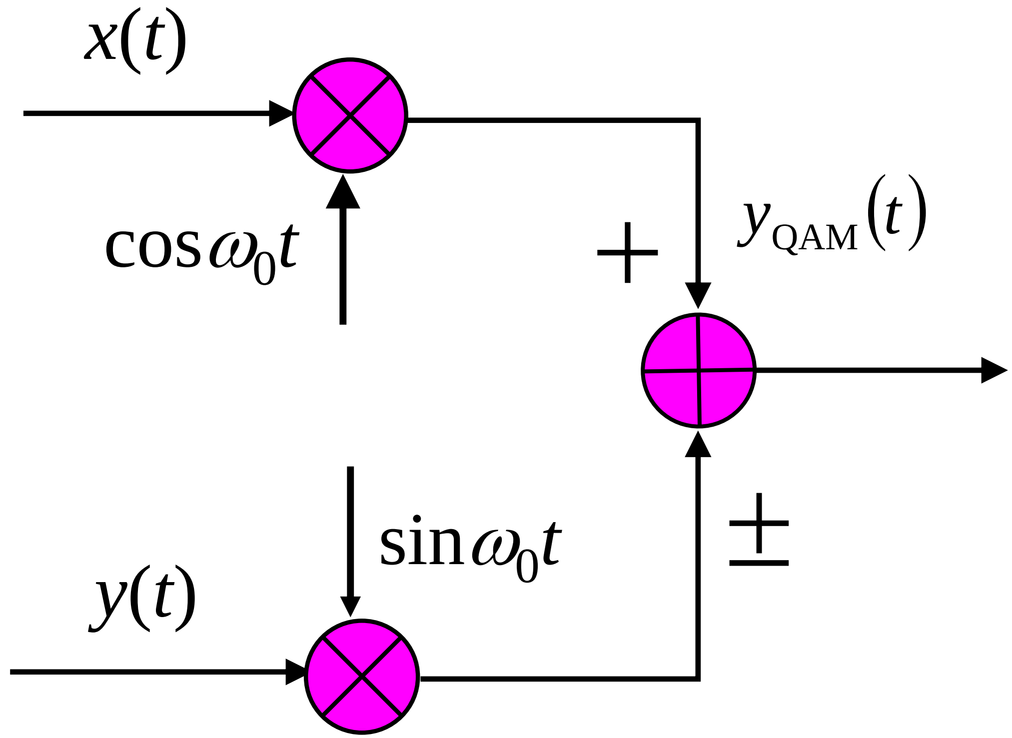
MEDIUM WAVES:
300 kHz ÷ 3 MHz (1 km ÷ 100 m)
AM Emissions:
522 kHz ÷ 1611 kHz
Distance covered:
~100 km
Propagation:
ground wave (incl. line of sight)

Amplitude Modulation -
- bandwidth efficient schemes



Quadrature Amplitude Modulation - QAM

$$y_{\mathrm{QAM}}(t) = x(t)\cos\omega_0 t \pm y(t)\sin\omega_0 t$$



Prove that it is possible to simultaneously coherently detect both signals *x*(*t*) and *y*(*t*).
Recommend the block diagram of the receiver.

AM power efficiency

Determine the power efficiency of the single tone AM signal:

$$y_{\mathrm{AM}}(t) = A_0 \left(1 + m \cos \omega_{\mathrm{m}} t\right) \cos \omega_0 t$$

<u>Power efficiency</u> is defined as the ratio of „information power" to the entire power of the AM signal.

**Hint: convert the AM signal to a sum of unmodulated carrier (which does not convey information)
and two sidebands (which convey information).**

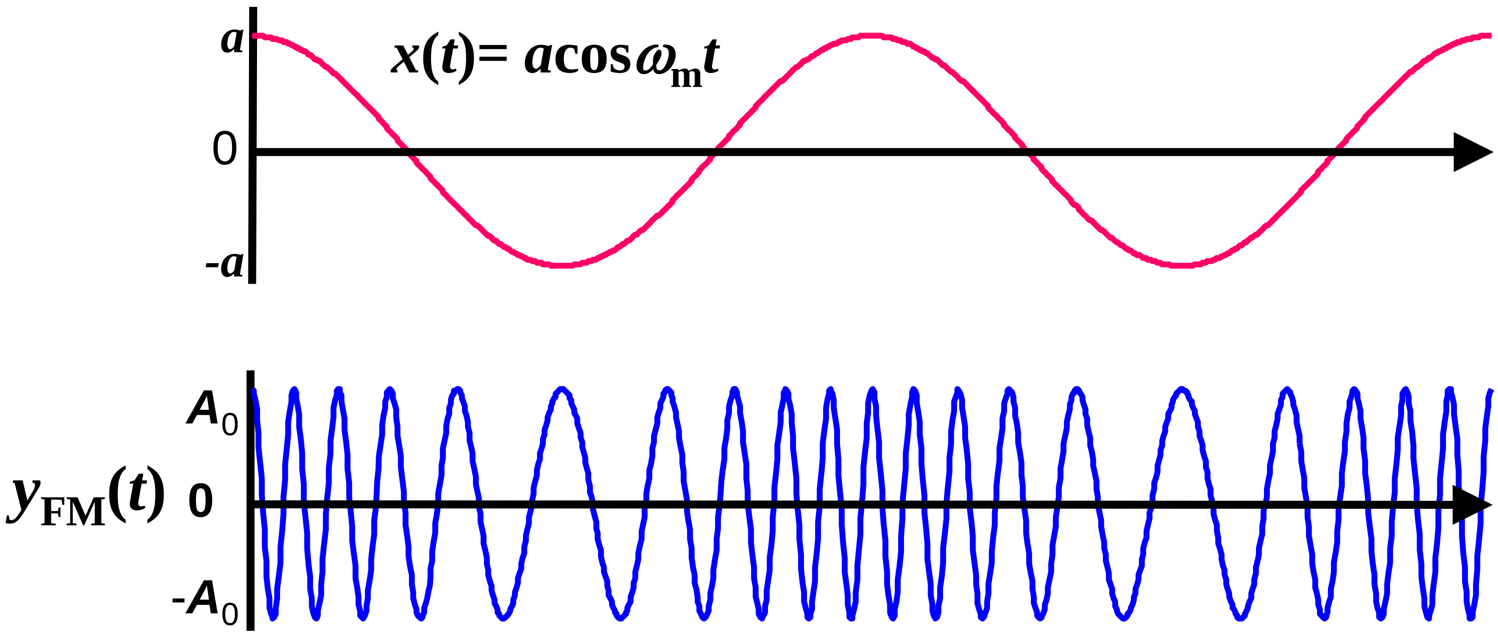
Frequency Modulation (FM)

$$\omega(t) \overset{df}{=} \omega_0 + kx(t)$$

$$c(t) = A_0 \cos \omega_0 t$$

$$y_{\mathrm{FM}}(t) \overset{df}{=} A_0 \cos\left[\omega_0 t + k\int_0^t x(\tau)d\tau\right]$$

k - modulator constant



Instantaneous frequency deviation $kx(t)$ from the carrier frequency ω_0 of the modulated signal fluctuates according to variations of the modulating signal.

Frequency Deviation and Modulation Index

Single Tone FM ($x(t) = a\cos\omega_{\mathrm{m}}t$)

$$y_{\mathrm{FM}}(t) \stackrel{df}{=} A_0\cos\left[\omega_0 t + k\int_0^t x(\tau)d\tau\right]$$

$$y_{\mathrm{FM}}(t) = A_0\cos(\omega_0 t + \underbrace{\frac{ka}{\omega_{\mathrm{m}}}}_{\Delta\varphi}\sin\omega_{\mathrm{m}}t)$$

$$y_{\mathrm{FM}}(t) = A_0\cos(\omega_0 t + \Delta\varphi\sin\omega_{\mathrm{m}}t)$$

$$\Delta\varphi = \Delta\omega/\omega_{\mathrm{m}}$$

Phase modulation index

$$\omega(t) = \omega_0 + kx(t)$$

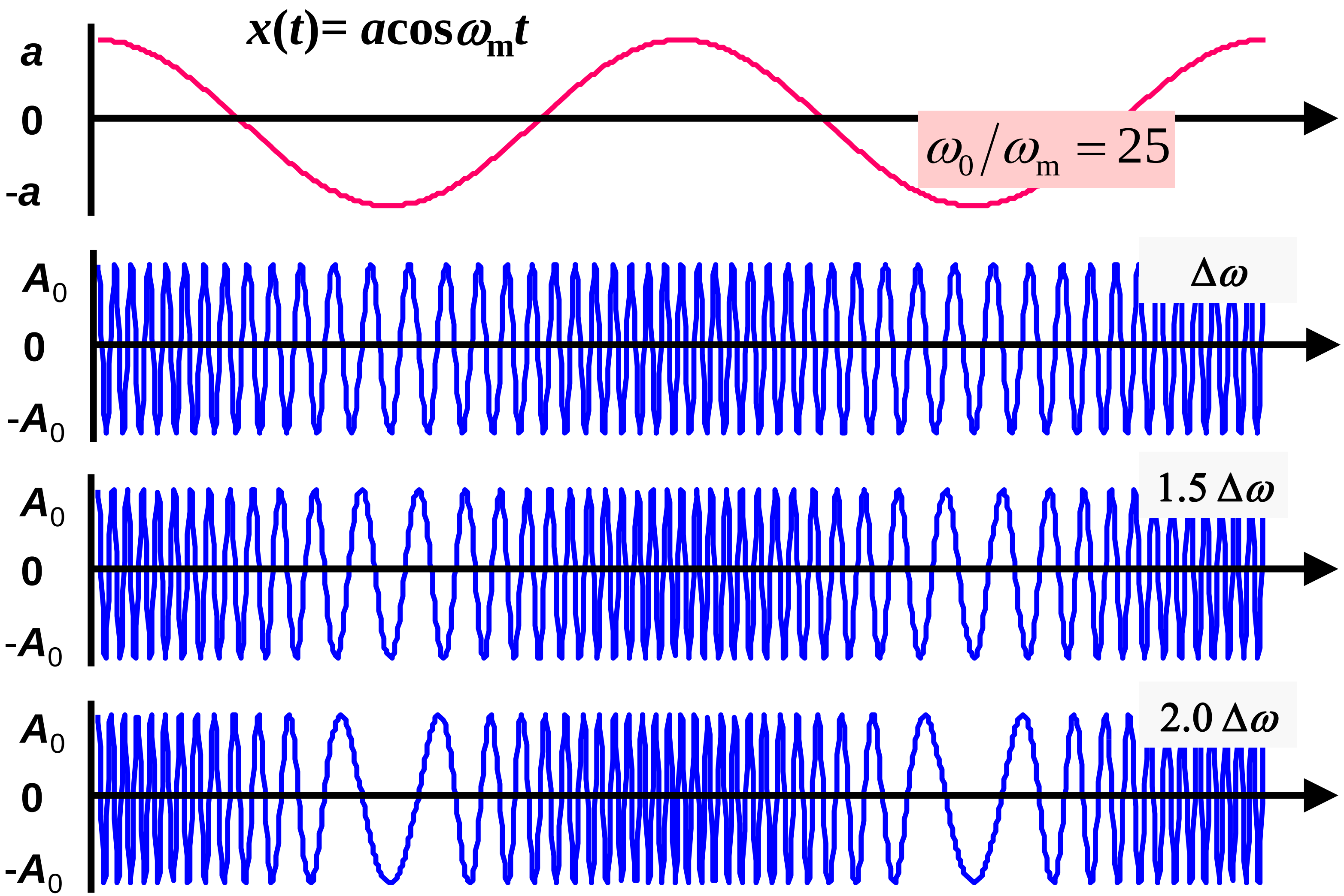
$$\omega(t) = \omega_0 + \underbrace{ka}_{\Delta\omega}\cos\omega_{\mathrm{m}}t$$

$$\Delta\omega = \Delta\varphi\times\omega_{\mathrm{m}}$$

Frequency deviation

FM frequency deviation index $\Delta\omega$ shows how much the FM instantaneous frequency $\omega(t) = \omega_0 + \Delta\omega\cos\omega_{\mathrm{m}}t$ is deviating around the carrier fequency ω_0. However, the FM process is a single parameter modulation due to the relations $\Delta\varphi = \Delta\omega/\omega_{\mathrm{m}}$.

FM Waves



$x(t) = a\cos\omega_m t$
a
0
-a
$\omega_0/\omega_m = 25$
A_0
0
$-A_0$
$\Delta\omega$
A_0
0
$-A_0$
$1.5\,\Delta\omega$
A_0
0
$-A_0$
$2.0\,\Delta\omega$

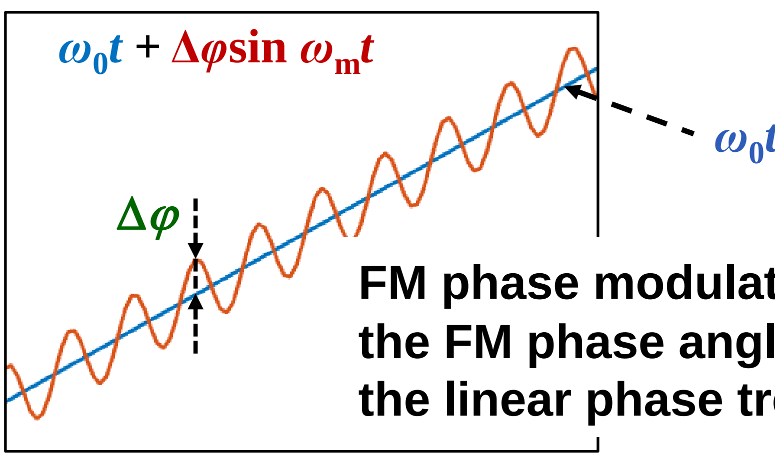
Frequency Deviation and Phase Modulation Index

Single Tone FM ($x(t) = a\cos\omega_{\mathrm{m}}t$)

$$\varphi_{\mathrm{FM}}(t) \overset{df}{=} A_0 \cos\left[\omega_0 t + k\int_0^t x(\tau)d\tau\right]$$

$$\varphi_{\mathrm{FM}}(t) = A_0 \cos(\omega_0 t + \underbrace{\frac{ka}{\omega_{\mathrm{m}}}}_{\Delta\varphi} \sin\omega_{\mathrm{m}} t) = A_0 \cos(\omega_0 t + \Delta\varphi \sin\omega_{\mathrm{m}} t)$$

FM phase modulation index



FM phase modulation index $\Delta\varphi$ shows how much the FM phase angle $\omega_0 t + \Delta\varphi\sin\omega_{\mathrm{m}} t$ is deviating around the linear phase trend $\omega_0 t$.

Spectrum Analysis of Single Tone FM

$$x(t) = a\cos\omega_{\mathrm{m}} t$$

$$y_{\mathrm{FM}}(t) = A_0 \cos(\omega_0 t + \underbrace{\frac{ka}{\omega_{\mathrm{m}}}}_{\Delta\varphi} \sin\omega_{\mathrm{m}} t)$$

$$y_{\mathrm{FM}}(t) = A_0 \cos(\omega_0 t + \Delta\varphi \sin\omega_{\mathrm{m}} t)$$

$$y_{\mathrm{FM}}(t) = A_0 \sum_{n=-\infty}^{+\infty} J_n(\Delta\varphi)\cos\left(\omega_0 + n\omega_{\mathrm{m}}\right)t$$

$J_n(\Delta\varphi)$ – Bessel's functions of the 1st kind, *n*-th order

Spectrum of a single tone FM is composed of spectral lines located at frequencies $\omega_0 \mp \omega_{\mathrm{m}}$; amplitudes of spectral lines depend on Bessel functions.

Spectrum Analysis of Single Tone FM

$$y_{\mathrm{FM}}(t) = A_0 \cos(\omega_0 t + \Delta\varphi \sin \omega_{\mathrm{m}} t)$$
$$= A_0 \,\mathrm{Re}\left\{ e^{j\Delta\varphi \sin \omega_{\mathrm{m}} t} e^{j\omega_0 t} \right\}$$

Find the exponential Fourier series of a single tone FM

Exponential Fourier Series

$$e^{j\Delta\varphi \sin \omega_{\mathrm{m}} t} = \sum_{n=-\infty}^{+\infty} J_n(\Delta\varphi) e^{jn\omega_{\mathrm{m}} t}$$

$J_n(\Delta\varphi)$ – **Bessel's functions of the 1st kind, *n*-th order**

Spectrum Analysis of Single Tone FM

$$e^{j\Delta\varphi \sin \omega_{\mathrm{m}} t} = \sum_{n=-\infty}^{+\infty} J_n(\Delta\varphi) e^{jn\omega_{\mathrm{m}} t}$$

$$J_n(\Delta\varphi) = \frac{\omega_{\mathrm{m}}}{2\pi} \int_{-\pi/\omega_{\mathrm{m}}}^{+\pi/\omega_{\mathrm{m}}} e^{j\Delta\varphi \sin \omega_{\mathrm{m}} t} e^{-jn\omega_{\mathrm{m}} t} dt =$$

$$= \frac{1}{2\pi} \int_{-\pi}^{\pi} e^{j(\Delta\varphi \sin \tau - n\tau)} d\tau; \quad \omega_{\mathrm{m}} t = \tau$$

$J_n(\Delta\varphi)$ – **Bessel's functions of the 1st kind, *n*-th order**

Spectrum Analysis of Single Tone FM

WORK IN PROGRESS

$$y_{\mathrm{FM}}(t) = A_0 \,\mathrm{Re}\left\{e^{j\Delta\varphi \sin\omega_{\mathrm{m}} t} e^{j\omega_0 t}\right\} = A_0 \left\{e^{j\omega_0 t} \sum_{n=-\infty}^{+\infty} J_n(\Delta\varphi) e^{jn\omega_{\mathrm{m}} t}\right\} =$$

$$A_0 \,\mathrm{Re}\left\{\sum_{n=-\infty}^{+\infty} J_n(\Delta\varphi) e^{j(\omega_0 t + n\omega_{\mathrm{m}} t)}\right\} = A_0 \sum_{n=-\infty}^{+\infty} J_n(\Delta\varphi) \cos(\omega_0 + n\omega_{\mathrm{m}}) t$$

$$y_{\mathrm{FM}}(t) = A_0 \sum_{n=-\infty}^{+\infty} J_n(\Delta\varphi) \cos(\omega_0 + n\omega_{\mathrm{m}}) t$$

Properties of Bessel's Functions

1) $J_n(\Delta\varphi) \in \mathcal{R}$

2) $J_{-n}(\Delta\varphi) = (-1)^n J_n(\Delta\varphi) \to |J_{-n}(\Delta\varphi)| = |J_n(\Delta\varphi)|$

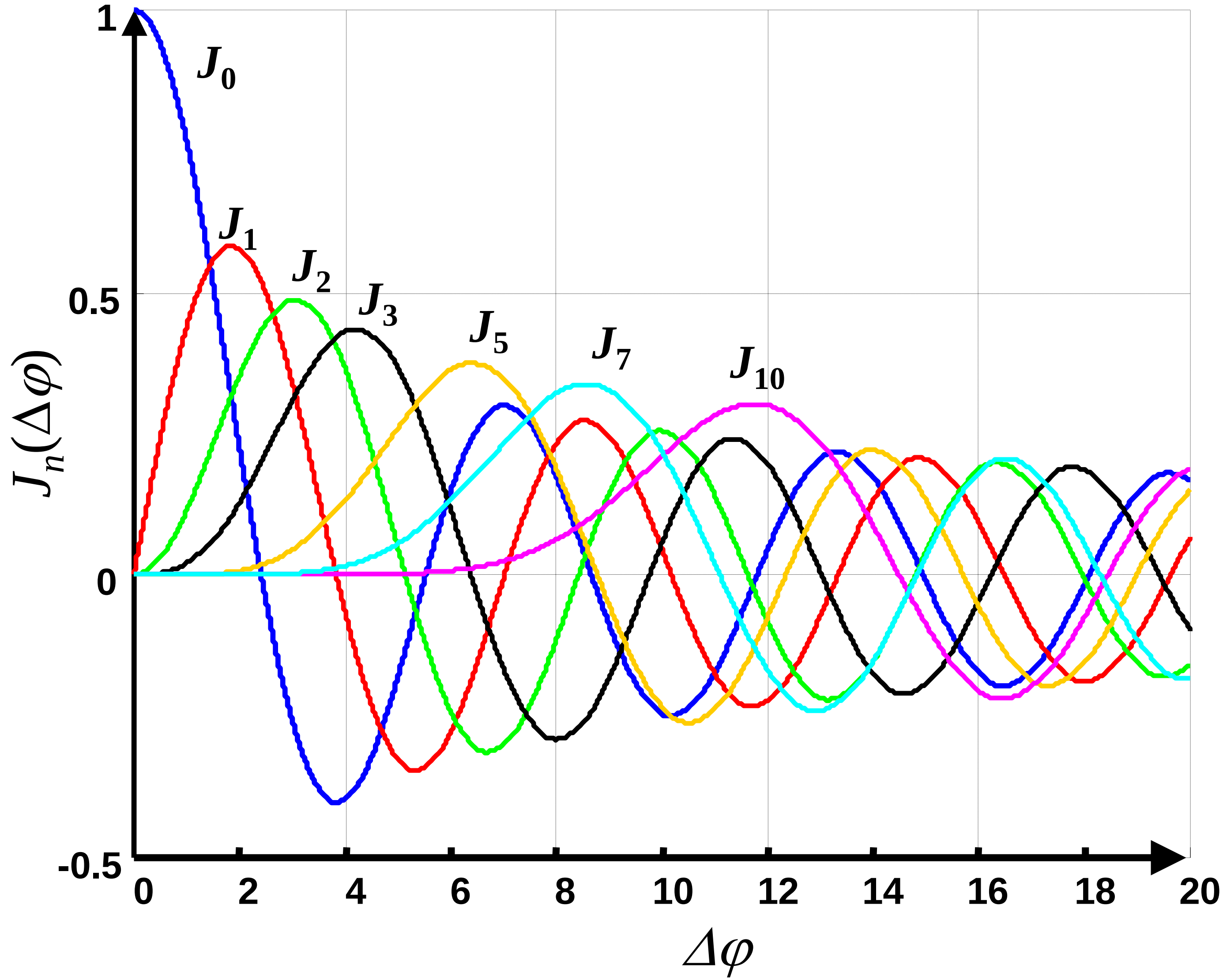
3) $J_n(\Delta\varphi) = \dfrac{2(n-1)}{\Delta\varphi} J_{n-1}(\Delta\varphi) - J_{n-2}(\Delta\varphi)$

4) $J_n(\Delta\varphi)$ are decreasing to zero $J_n(\Delta\varphi) \to 0$ for $\Delta\varphi \to \infty, n \to \text{const}$

5) $J_n(\Delta\varphi)$ are decreasing to zero $J_n(\Delta\varphi) \to 0$ for $\Delta\varphi = \text{const}, n \to \infty$

Properties 4 and 5 are important when looking for a bandwidth of the single tone FM modulation (Caeson's rule).

Plots of Bessel's Functions



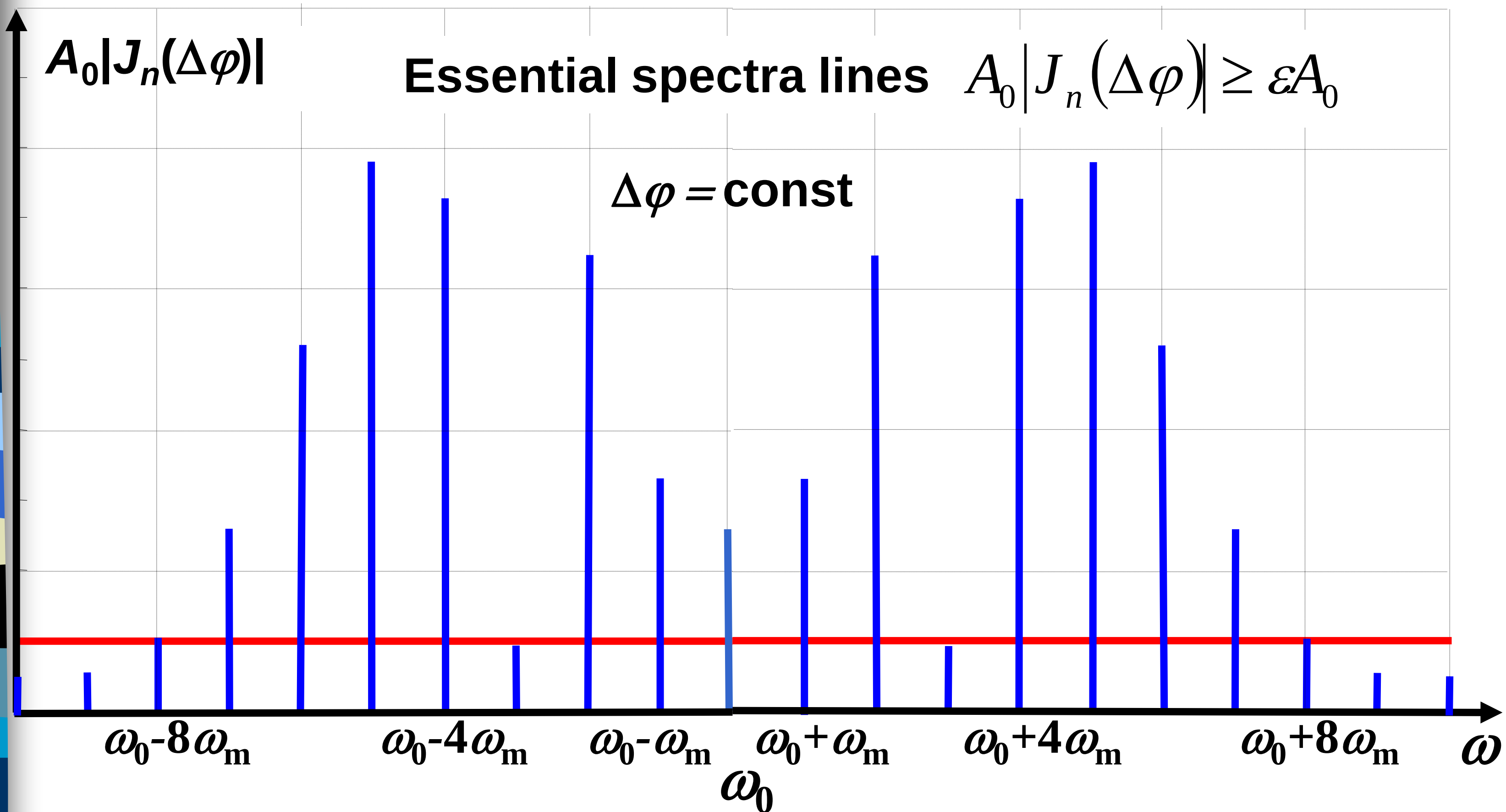
Values of Bessel's Functions

$\Delta\varphi$	n												
	0	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**
0	1,00												
1	0,77	0,44	0,11	0,02									
2	0,22	0,58	0,35	0,13	0,03	0,01							
3	-0,26	0,34	0,49	0,31	0,13	0,04	0,01						
4	-0,40	-0,07	0,36	0,43	0,28	0,13	0,05	0,02					
5	-0,18	-0,33	0,05	0,36	0,39	0,26	0,13	0,05	0,02	0,01			
6	0,15	-0,28	-0,24	0,11	0,36	0,36	0,25	0,13	0,06	0,02	0,01		
7	0,30	0,00	-0,30	-0,17	0,16	0,35	0,34	0,23	0,13	0,06	0,02	0,01	
8	0,17	0,23	-0,11	-0,29	-0,11	0,19	0,34	0,32	0,22	0,13	0,06	0,03	0,01
9	-0,09	0,25	0,14	-0,18	-0,27	-0,06	0,20	0,33	0,31	0,21	0,12	0,06	0,03
10	-0,25	0,04	0,25	0,06	-0,22	-0,23	-0,01	0,22	0,32	0,29	0,21	0,12	0,06

$J_n(\Delta\varphi)$

Each function $J_n(\Delta\varphi)$ attains its maximum value for some modulation index $\Delta\varphi$ and then is asymptotically tending to 0, $J_n(\Delta\varphi)\to 0$.

Line spectrum of a single tone FM



Line spectrum $A_0|J_n(\Delta\varphi)|$ is an even function as $|J_{-n}(\Delta\varphi)| = |J_n(\Delta\varphi)|$ around the carrier frequency ω_0.

Spectral lines beyond some critical *n* value are decresing to zero.

FM frequency bandwidth

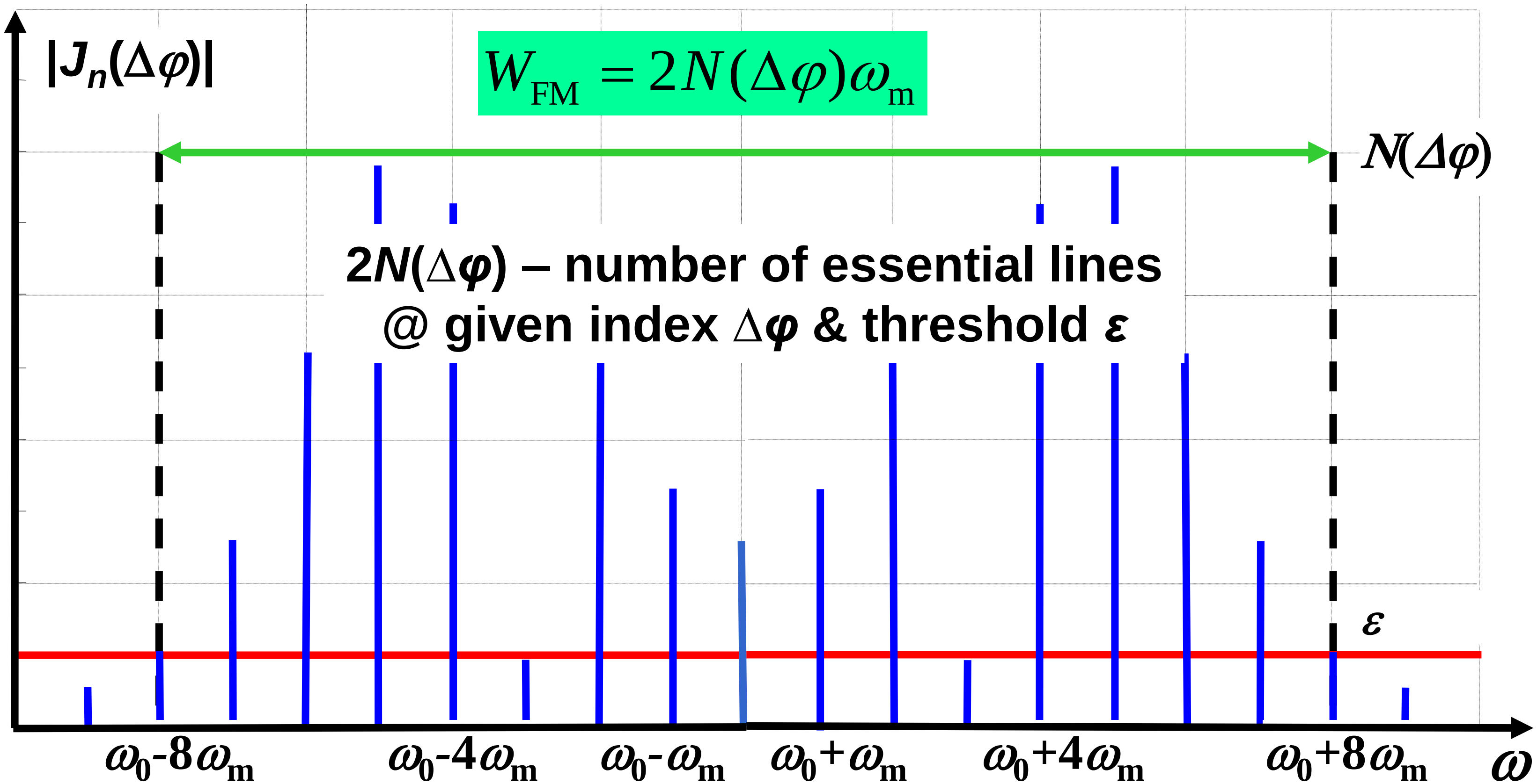
$$\varphi_{\mathrm{FM}}(t) = A_0 \sum_{n=-\infty}^{+\infty} J_n(\Delta\varphi)\cos\left(\omega_0 + n\omega_{\mathrm{m}}\right)t$$

Essential lines

$$A_0\left|J_n(\Delta\varphi)\right| \geq \varepsilon A_0$$

$$\left|J_n(\Delta\varphi)\right| \geq \varepsilon$$

$$\varepsilon = 0.01; .0.05; 0.1$$



Spectrum of FM (Carson's rule)

$$W_{\mathrm{FM}} = 2N(\Delta\varphi)\omega_{\mathrm{m}}$$

where:

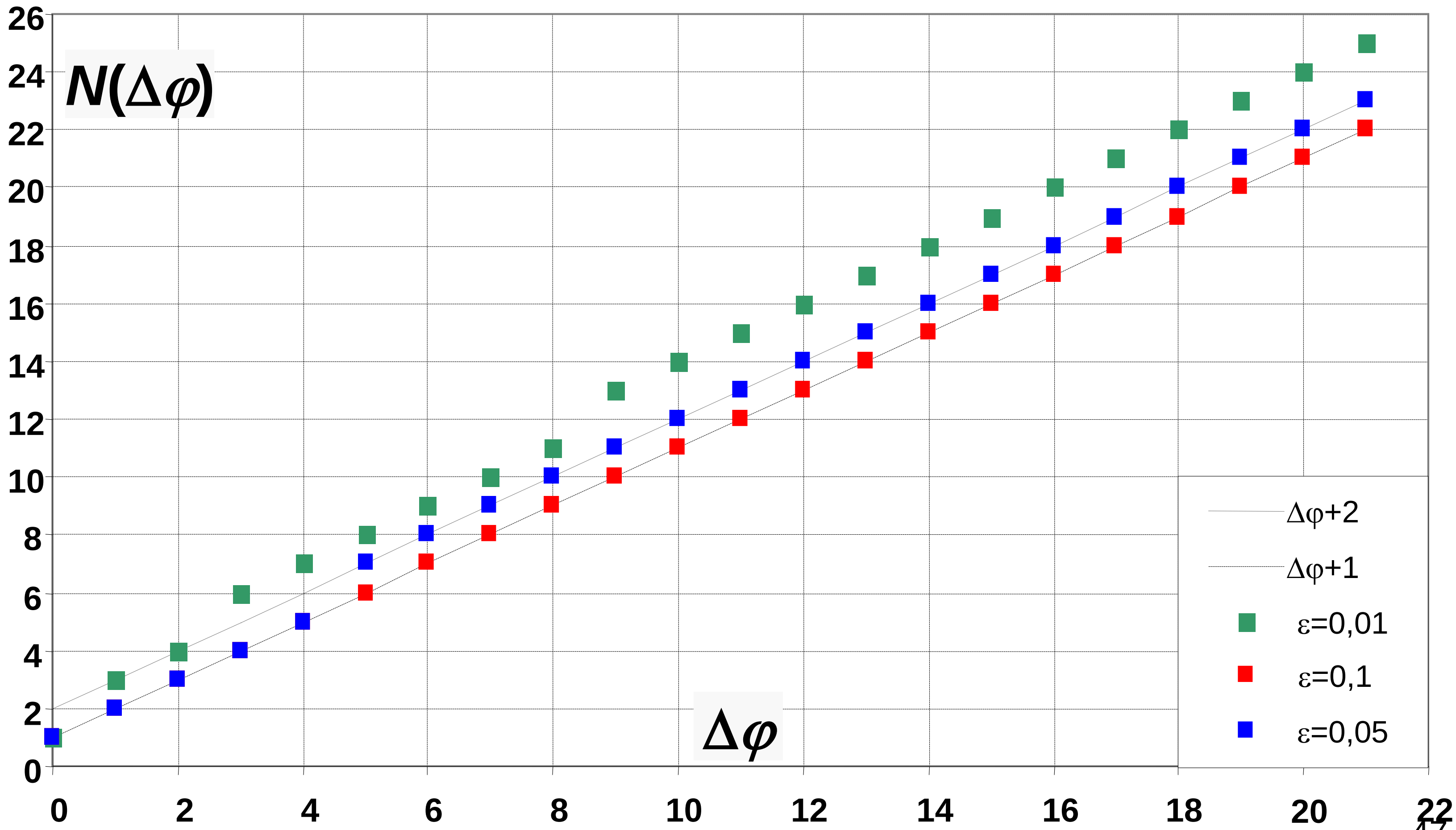
$$N(\Delta\varphi) \approx \Delta\varphi + 1$$

is the number of „essential" spectra lines, so:

$$W_{\mathrm{FM}} = 2(\Delta\varphi + 1)\omega_{\mathrm{m}}$$
$$W_{\mathrm{FM}} = 2(\Delta\omega + \omega_m)$$

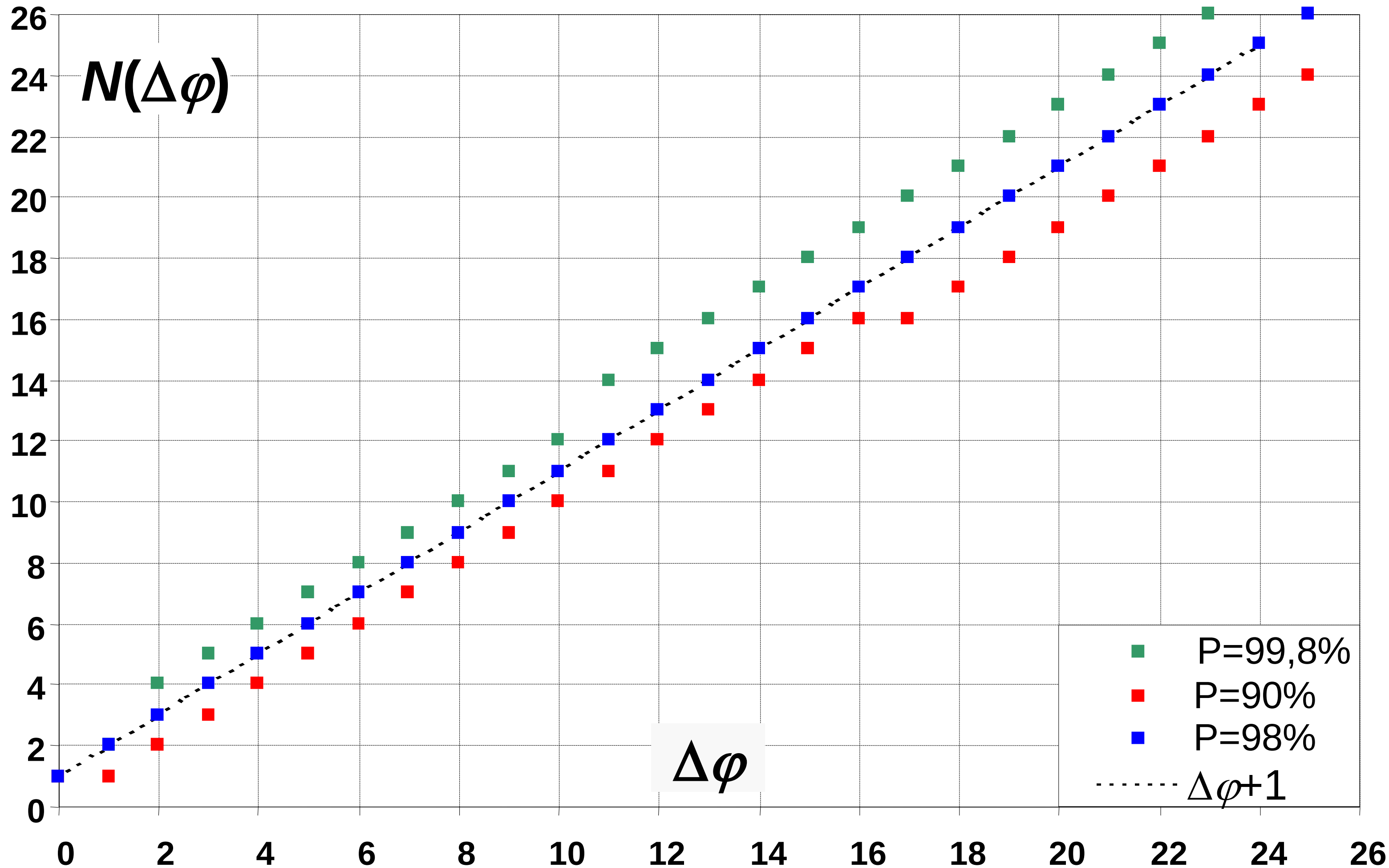
essential spectrum lines N(Δφ)

The spectrum line is considered to be essential whenever its amplitude exceeds some predefined level εA_0, $0{,}01 < \varepsilon < 0{,}1$.



essential spectrum lines N(Δφ)

The number of essential spectrum lines is derived from the fractional power of the FM signal.



Spectrum of FM – rule of thumb

$$W_{\mathrm{FM}} = 2N(\Delta\varphi)\omega_{\mathrm{m}}$$
$$N(\Delta\varphi) = \Delta\varphi + 1$$

For:

$$\Delta\varphi >> 1$$
$$N(\Delta\varphi) \approx \Delta\varphi \approx \Delta\omega/\omega_{\mathrm{m}}$$

so the FM bandwidth equals (a rule of thumb):

$$W_{\mathrm{FM}} = 2\Delta\omega$$
$$\kappa_{\mathrm{FM}} = 2\Delta\omega/\omega_{\mathrm{m}} = 2\Delta\varphi$$

FM stereo audio broadcasting

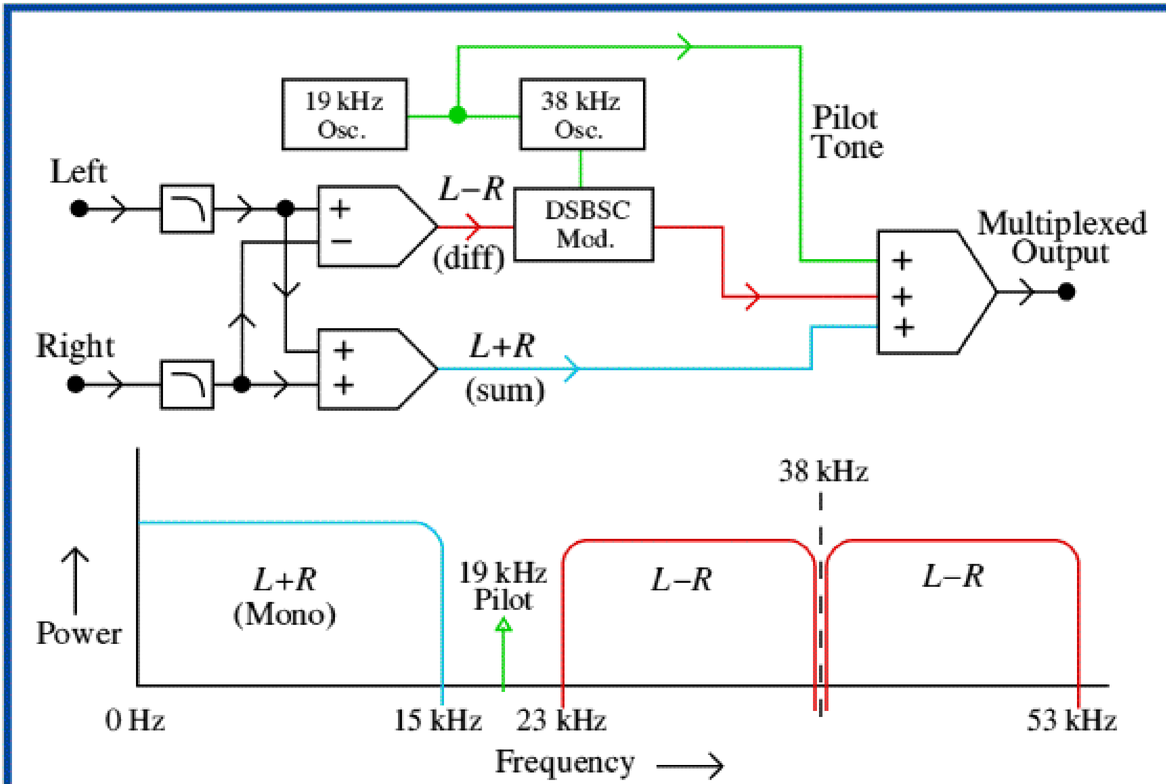
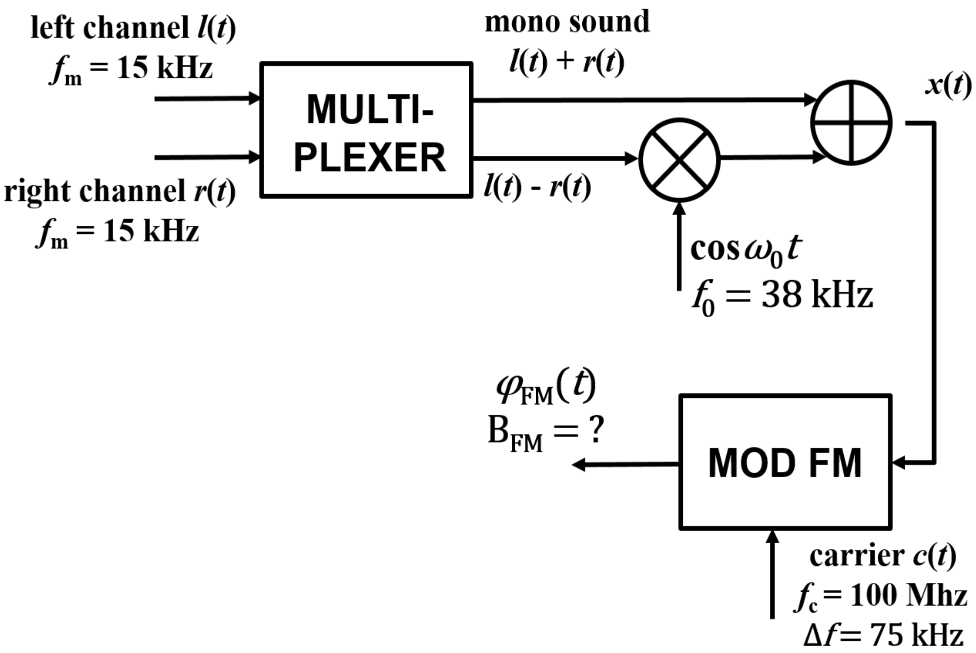


Figure 21.1 FM Stereo Multiplex signal.

$$\Delta f = 75\,\mathrm{kHz},\ f_{\mathrm{m}} = 53\,\mathrm{kHz}, \Delta\varphi = 1.42$$
$$B = 2(1+1.42)\times 53\,\mathrm{kHz} \cong 256\,\mathrm{kHz}$$

Find the bandwidth of the stereo audio FM signal.

FM stereo audio broadcasting



The block diagram presents how the modulating signal *x*(*t*) for the FM stereo audio broadcasting is produced.

1. Sketch the signal *x*(*t*) for for two arbitrary signals *l*(*t*) and *r*(*t*).

2. Sketch the spectrum *x*(*t*) ↔ *X*(*ω*) for two arbitrary signals *l*(*t*) and *r*(*t*

3. Find the bandwidth of the FM signal.

4. Propose a block diagram of the FM stereo sound receiver.

Armstrong Modulator

WORK IN PROGRESS

Prove that it is possible to generate the single tone FM modulation $\varphi_{\mathrm{FM}}(t) = A_0 \cos[\omega_0 t + \Delta\varphi \sin\omega_{\mathrm{m}} t]$ **for a small modulation index** $\Delta\varphi \approx 0$ **using the DSB amplitude modulation. For this purpose use a complex representation** $\varphi_{\mathrm{FM}}(t) = A_0 \mathrm{Re}\{\exp j(\omega_0 t + \Delta\varphi \sin\omega_{\mathrm{m}} t)\} = A_0 \mathrm{Re}\{\exp(j\omega_0 t)\exp(j\Delta\varphi \sin\omega_{\mathrm{m}} t)\}$**. 1. Approximate the component responsible for a modulation effect using the approximation** $\exp(z) \approx 1 + z$ **for** $z \approx 0, \mathrm{z} \in \boldsymbol{C}$**. 2. Determine the approximate signal** $\varphi_{\mathrm{FM}}(t)$ **and explain its difference to the AM signal. 3. How much is the maximum modulation index** $\Delta\varphi$ **for the signal** $\varphi_{\mathrm{FM}}(t)$ **having two essential spectrum lines? Hint: Bessel functions may be calculated with the page https://pinecalculator.com/bessel-function-calculator.**

FM demodulation by an envelope detector

Prove that the tandem connection of a differentiator and an envelope detector makes possible to properly demodulate the FM signal.

PM demodulation by a coherent detector

Let us consider a PM signal with a modulating signal $|x(t)| \leq 1$. Find conditions allowing for an approximate coherent detection of the PM signal.